Estimating the term structure of interest rates

In commenting on capital market rates for different maturities, the Bundesbank will in future use the (estimated) term structure of interest rates. This will replace the previous presentation by approximation in the form of (estimated) yield curves. In principle, the term structure of interest rates allows a more precise presentation and analysis of expectations in the bond market and ensures enhanced international comparability of the estimation results. Moreover, (implied) forward rates can be calculated directly from the term structure of spot rates. The following article explains the method used for estimating the term structure.

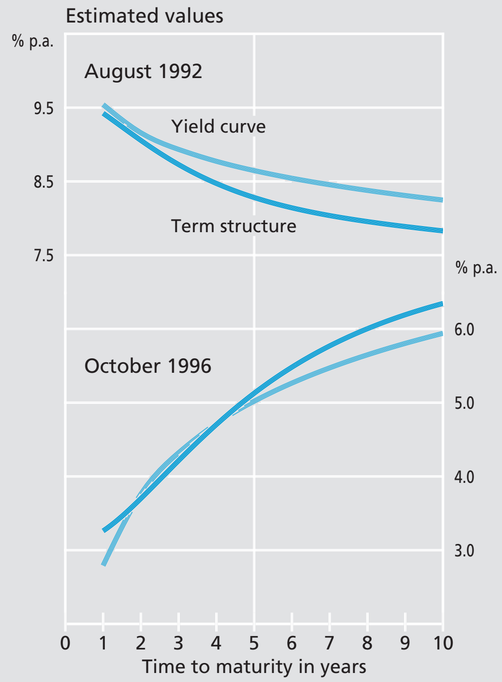
Term structure and yield curve

The term structure of interest rates shows the relation between the interest rates and maturities of zero-coupon bonds without risk of default. In the monetary policy context, it is primarily of interest as an indicator of the market's expectations regarding interest rates and inflation rates. Its slope can provide information about the expected changes in interest rates or inflation rates. Hitherto, this constellation was captured by way of approximation in the publications of the Deutsche Bundesbank by an (estimated) yield curve. From now on that approach is to be replaced by a direct estimation of the term structure of interest rates.[1] This approach is being adopted increasingly in the international context. In principle, it allows a more precise presentation and analysis of expectations and ensures enhanced cross-country comparability of the estimation results. Moreover, (implied) forward rates can be calculated directly from the term structure of (spot) rates. Although such forward rates contain the same information as the term structure of interest rates, in principle they make it easier to separate expectations for the short, medium and long term. The method used for estimating the term structure is explained below.[2]

Conceptual differences between interest rates and yields-to-maturity

The rate of return of a capital market investment corresponds to the (annual) rate of return which results from the relation between the redemption value and the current price. The calculation of the interest rate is simple in the case of debt securities which provide only one payment – such as zero-coupon bonds. But if several payments accrue during the debt security's life – as in the case of coupon bonds, which are customary in Germany – the rate of return on the individual payments may differ, depending on the time of payment. Whereas in calculating the yield-to-maturity all payment flows are discounted to current values at the same rate – i.e. the yield-to-maturity – in estimating the term structure of interest rates each payment flow is discounted at an interest rate which, depending on the reinvestment date and period, is to be expected according to the current market situation. Interest rates and yields-to-maturity of coupon bonds are only identical if a constant discount rate applies to all maturities, in other words if a horizontal term structure exists. In this case the assumption of reinvestment on which the calculation of yields-to-maturity is based is not a constraint. However, if, for example, interest rates rise with increasing maturity, this rise is underestimated by the yield curve. This means that the yield curve is below the term structure if the latter has a positive slope. The opposite

Term structure and yield curve in the nineties



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1 The results of the yield curve estimation which have been published hitherto in table II.7e in the Statistical Supplement to the Monthly Report 2, Capital market statistics are being replaced as from October 1997 by the results of the estimation of the term structure. The yield curve estimates will continue to be available to interested parties on request. For the procedure used by the Bundesbank to estimate the yield curve see: Deutsche Bundesbank, Interest rate movements and the interest rate pattern since the beginning of the eighties; Annex: Notes on the interpretation of the yield curve, Monthly Report, July 1991, pages 40–42.
2 For a detailed description see: Schich, S.T., Estimating the German term structure, Discussion paper 4/97, Economic Research Group of the Deutsche Bundesbank, to be published shortly.

applies in the case of a falling yield curve (see chart on page 62). This can complicate the analysis and interpretation of yield curves for monetary policy purposes. Such problems are avoided by the use of a term structure.

Estimating a continuous term structure ...

A continuous term structure would be observable directly in the bond market if a quotation for a (default) risk-free zero-coupon bond existed for each maturity. In reality, however, there are only a small number of such bonds and hence of observations. Admittedly, Federal bonds have a negligible default risk and hence come very close to the ideal of (default) risk-free bonds. But they are mostly coupon bonds.[3]

... from coupon bonds

The prices of zero-coupon bonds can be used to calculate the interest rates for the respective maturities relatively easily since the latter are the only unknown variable in the price equations of the bonds. This is not possible for coupon bonds (if the time to maturity is more than one year) since payments accrue at different points of time. To facilitate the calculation of interest rates, these individual payments have to be discounted not at constant, but – as mentioned – at maturity-related interest rates. The equation for the price of the coupon bond thus contains several unknown variables. For that reason the interest rates have to be calculated iteratively. Theoretical yields-to-maturity are calculated from a pre-specified term structure and compared with the observed yields on bonds outstanding. The (theoretical) term structure is then varied until the theoretical yields-to-maturity are (largely) identical with the actually observed yields on bonds outstanding.

Estimation approach

As in the case of estimating continuous yield curves, an assumption about the functional relationship between interest rates and maturities has to be made when estimating continuous term structures from the yields-to-maturity of coupon bonds. This decision is determined by the purpose for which the estimations are to be used. Basically, a trade-off exists between the "smoothness" of the estimated curve, on the one hand, and its flexibility, i.e. obtaining a maximum approximation to the observed data, on the other hand. For the purpose of monetary policy analysis, the approach developed by Nelson and Siegel and extended by Svensson is a good compromise.[4] This extended approach defines the interest rate as the sum of a constant and various exponential terms (in which the time to maturity has a negative sign in the exponent) and as a function of a total of six parameters:

$$
\begin{aligned}
z(T,\beta) = \beta_0 &+ \beta_1 \left(\frac{1-\exp(-T/\tau_1)}{(T/\tau_1)}\right) \\
&+ \beta_2 \left(\frac{1-\exp(-T/\tau_1)}{(T/\tau_1)} - \exp\left(-\frac{T}{\tau_1}\right)\right) \\
&+ \beta_3 \left(\frac{1-\exp(-T/\tau_2)}{(T/\tau_2)} - \exp\left(-\frac{T}{\tau_2}\right)\right).
\end{aligned}
$$

3 The splitting and separate trading of the principal and interest coupon ("stripping") was introduced for selected Federal bonds in July 1997. In principle, stripping creates a variety of additional securities which have the character of zero-coupon bonds. But the liquidity of such securities, and hence the information content of their prices compared with traditional coupon bonds, is rather low, at least at present.

4 Nelson, C.R. and Siegel, A.F. (1987), Parsimonious modeling of yield curves, Journal of Business, 60, 4, pages 473–489, and Svensson, L.E.O. (1994), Estimating and interpreting forward interest rates: Sweden 1992–94, IMF Working Paper 114, September.

Here $z\ (T,\ \beta)$ denotes the interest rate for maturity T as a function of the parameter vector β. β_0, β_1, β_2, β_3, τ_1 and τ_2 denote the parameters of this vector that are to be estimated. The functional form originally suggested by Nelson and Siegel does not contain the last term; β_3 is thus constrained to zero. Svensson's extension of this approach allows an additional turning point in the estimated curve.

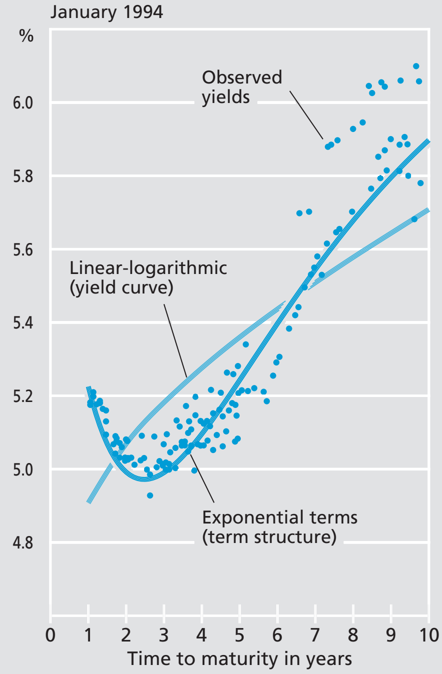
Our own calculations on data for the German bond market showed that the specification according to Svensson produces more favourable estimation statistics in some cases than the Nelson and Siegel approach. In other situations, however, the Svensson specification can be overparameterised. In such cases, the restricted form according to Nelson and Siegel suffices; but this problem has virtually no effect on the estimation results. For this reason, the Bundesbank publications will use the Svensson specification, especially since this facilitates better international comparability of the results.

Sufficient flexibility ...

The parametric approach using exponential terms is – both in its original formulation by Nelson and Siegel and in its extension by Svensson – sufficiently flexible to reflect the data constellations observed in the market. These include monotonically rising or falling, U-shaped, inverted U-shaped and S-shaped curves, some of which could not be captured by the linear-logarithmic regressions used previously. The greater flexibility of the approach featuring exponential terms compared with the linear-logarithmic approach is illustrated by the chart on this page showing the data constellation of January 1994. The pronounced U-shape which can be observed in the data is depicted well, whereas the linear-logarithmic approach generates a monotonically rising curve.

Comparison of different estimation approaches

January 1994



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... but smooth slope, providing information for monetary policy

Unlike non-parametric approaches, the estimation procedure described above smooths out individual kinks in the curve, so that the estimation results are relatively less dependent on individual observations. For that reason they are less suited to identify, say, abnormalities in individual maturity segments or in individual bonds, but they produce curves which are relatively free of outliers and thus easier to interpret for the purpose of monetary policy analysis. Moreover, the specification allows plausible extrapolations to be made for the segments extending beyond the observed maturities. The extrapolated long-term interest rates converge towards the value of the constant β_0, since the contribution of the exponential terms approaches zero with increasing maturity. The limit can be seen as the very long-term interest rate. On the other hand, non-parametric estimation approaches, or approaches which include terms that are linearly linked to the maturity (such as the linear-logarithmic approach), can produce implausible estimation values in long-term extrapolations, such as negative or infinitely high interest rates.

Daily estimation from prices of Federal securities

The parameters of the above-mentioned function are estimated daily. The estimations are based on the prices of Federal bonds, five-year special Federal bonds and Federal Treasury notes with a (time to) maturity of at least three months. These securities are largely homogeneous, and the maturity range of up to ten years, which is at the focus of interest, is sufficiently well represented. The parameters are calculated using a non-linear optimisation procedure. The optimisation criterion applied is the minimisation of the squared deviations of the estimated yields-to-maturity (or of the yields calculated from the theoretical prices) from the observed yields (or from the yields calculated from the observed prices). Yield errors are minimised rather than price errors, since the focus is on estimating interest rates and not prices and because the minimisation of price errors may be associated with relatively large yield deviations for bonds with a short (time to) maturity. The specification of constraints for some parameters ensures that the estimated interest rates are positive (e.g. the constraint that β_0, τ_1 and τ_2 are greater than zero in the Svensson approach) and that the calculations using historical data invariably produce plausible curve shapes.

Forms of presentation

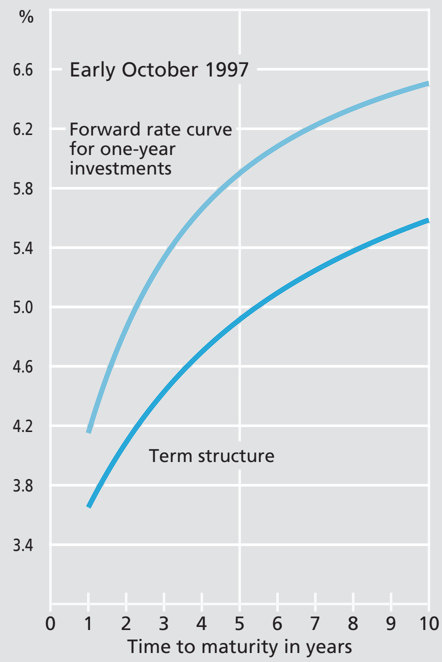
Usually, the relationship between the maturity and interest rates is depicted in the form of a term structure of (spot) interest rates; starting from the present, it shows the interest rates on investments for a variety of maturities. From the term structure (assuming an arbitrage equilibrium between the different maturity segments) the "implied" rate of return on future investments – based on present market conditions – can also be derived. These rates are called implied forward rates, since they cannot be observed directly and because they show the rate of return on forward transactions. Whereas, for example, the ten-year (spot) rate indicates the rate of return over ten years as measured from today, the one-year forward rate in nine years' time shows the return on a one-year bond in the tenth year. The forward rate curve shows the returns on a succession of future capital market investments (assuming one-year investments, as a rule). It will be above (below) the term structure of interest rates if the latter rises (falls). This is demonstrated by the chart on page 66.

Term structure, forward rate curve and expectations hypothesis

According to the expectations hypothesis of the term structure of interest rates, a financial investment yields the same expected return for a given period, irrespective of whether a succession of short-term investments is made or a single longer-term investment is made. Under this precondition, the one-year (implied) forward rate corresponds to the one-year (spot) interest rate expected for the same period. In this case, the slope of the term structure, measured as the difference between interest rates for various maturities, provides information about the expected average changes in short-term interest rates over the corresponding period. By contrast, the shape of the forward rate curve directly shows the expected future course of (spot) interest rates. This is interesting from a monetary policy point of view, since it allows a better separation of expectations over the short, medium and long term than the term structure does. However, the objections raised against an overly strict interpretation of the term structure in the sense of the expectations theory apply even more forcefully to the forward rate curve; in the first place, the existence of risk and forward premiums which vary over time should be mentioned, as they can heavily affect the implied forward rates. Since corresponding empirical studies have generally been unable to reject the existence of such time-variable premiums, the forward rate curve should be interpreted with particular caution.

Term structure
and foward rate curve

Estimated values



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