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Chapter 6

Arbitrage and Hedging with Options

INTRODUCTION

In chapter 5 we have explored arbitrage and hedging with spot and forward contracts and derived profit measures under different strategic scenarios. In this chapter, we study the feasibility and opportunity of arbitrage with the underlying cover of currency options. Theoretical exploration and market analysis of derivative securities have colored the landscape of financial economics and investment practice over the past two decades. Ever since the publication of the seminal papers by Black and Scholes (1972, 1973) and Merton (1973), and a few years later by Cox, Ross, and Rubinstein (1979), analysts and practitioners have virtually revolutionized investment strategies in financial markets worldwide. Options, futures, forward, swaps, and so on—in their simple and synthetic forms—have provided various opportunities for covering risk and increasing speculative interests. Following the lead of earlier research, Garman and Kohlhagen (1983), Biger and Hull (1983), Giddy (1983), Grabbe (1983), and Yang (1985), to name a few, have extended options in the domain of foreign currencies. A rich and extremely useful literature, for example, Geske and Johnson (1984), Brennan (1979), and Black (1975), has been developed on the way—examining, enunciating, and illustrating many aspects of investment through the use of derivative securities. Merton, Scholes, and Gladstein (1978) provide a simulation of risk and returns of alternative-option portfolio strategies. Mueller (1981), Pounds (1978), and Pozen (1978), Grube, Panton, and Terrell (1979), Jones (1984), Yates and Kopprasch (1980), among others, examine covered call-and-put options and other facets of these instruments in investment strategies.

Options, futures, and other derivative securities have been extensively used as a consequence to hedge and/or speculate in financial markets, and the scope of the use of these securities has been the subject of continuous studies.

In this chapter, we attempt to use currency options—puts and calls—to derive conditions under which a trader can make profits from his trading strategy without assuming any risk, and how he can compound his profits via option-covered arbitrage upon iterative plays in the marketplace. For the derivation of the basic results, we first simplify the analytical structure by assuming away transaction costs. Once the results are derived in this simplified structure, we bring out transactions costs by way of introducing *ask* and *bid* quotes in the foreign exchange rates, and *deposit* and *borrowing* rates of interest that banks give and charge when investors deposit their funds in banks and borrow funds from banks, respectively.

In the existing literature, arbitrage operations in foreign exchange markets have been defined conspicuously with spot and forward contracts in terms of foreign and domestic interest rates. The classic research in this area must be attributed to Frenkel and Levich (1975) and Deardorff (1979). Using proportional costs of transactions, Frenkel and Levich (1975) show that interest-rate parity is bound by a neutral band, which is defined by:

$$\left[\frac{F - S\left(\frac{1+r}{1+r^*}\right)}{S\left(\frac{1+r}{1+r^*}\right)}\right] = t + t^* + t_s + t_f,$$

where S, F = spot; forward rate of exchange r, r^* = domestic; foreign rate of interest t, t^* = proportional transaction costs in domestic and foreign money markets; and t_s, t_f = proportional transaction costs in spot and forward exchange markets, respectively. Deardorff (1979) illustrates that in one-way arbitrage, the neutral band becomes narrower, as shown below:

$$\left[\frac{F - S\left(\frac{1+r}{1+r^*}\right)}{S\left(\frac{1+r}{1+r^*}\right)}\right] = t + t^* - t_s - t_f.$$

Callier (1981), Bahmani-Oskooee and Das (1985), and Clinton (1988) illustrate further narrowing of the neutral band under different market conditions. Rhee and Chang (1992) attempt to measure arbitrage profitability and its persistence over time in the foreign currency market with spot and forward contracts. More recently, Blenman (1992a, 1992b), Blenman and Thatcher (in press), and Ghosh (in press-a, in press-b) have examined arbitrage profitability and the minimum and maximum bounds for such profits under conditions of market imperfection where market imperfection is captured directly by bid and ask quotes in the foreign exchange market, and deposit and borrowing rates in money markets (as opposed to proportional transaction costs in those markets as envisaged in earlier works).[1]

In this chapter, however, we move away from both of the approaches in the existing literature. Instead, we introduce options in the currency market as the instruments to cover risk in arbitrage activities, then analyze the feasibility of profitable arbitrage and the compounding of derivable profits under alternative operational frameworks. In the sections dealing with option covered currency without transaction costs, we enunciate the conditions under which arbitrage with option cover available in the market place admits of profits and the compounding of that initial profit level to a rational and active investor. Three distinct investment scenarios and measures of profits are presented in this section. The first scenario depicts what can be termed pure arbitrage profits when the investor plays with the profits alone (generated in the initial stage) from round two onward. The second scenario extends the analytical framework envisioned in the first scenario to churn in pure arbitrage profits with full utilization of all the funds that are potentially available to the investor. In the third scenario we extend the second scenario with further appeal to practical notion of cost consideration. The following sections introduce transaction costs by way of bringing out ask and bid quotes of the foreign exchange rates, deposit and borrowing rates of interest in the money market, and brokerage, fee-inclusive option premiums.

OPTION COVERED CURRENCY ARBITRAGE WITHOUT TRANSACTION COSTS

Before we begin to develop the analytical vehicle, let us introduce the notations that will be used in this section:

S = spot rate of exchange (of, say, 1 French franc in terms of U.S. dollar(s));
X_P = exercise price of 1-month put option;
P = put premium (as observed in the market);
X_C = exercise price of 1-month call option;
C = call premium (as observed in the market);
r = domestic interest rate for one month;
r^* = foreign interest rate for one month.

Pure Arbitrage Profits with Newly Created Profits

Consider an investor who has M dollars at his disposal (or can borrow M amount from his bank at the interest rate of r) to begin investing in the currency market. He converts his M dollars into, say, French francs at the spot rate and gets M/S French francs, which he can invest in the French market at r^*, and thus make $(M/S)(1 + r^*)$ at the end of one month. There is no risk involved in the French franc amount. This investor who starts off with M dollars can hedge his position by exercising his put option (which we assume he has bought) by selling $(M/S)(1 + r^*)$ French francs at the rate X_p, thus retrieving $(M/S)(1 + r^*)(X_p - P(1 + r))$ amount in U.S. dollars. Since the investor has purchased the put option for P, which at the end of one month equals $P(1 + r)$, and his original amount of investable funds M dollars are worth $M(1 + r)$, his profits at the end of one month, in this instance, is then measured by π_1:[2]

$$\pi_1 = \frac{M}{S}(1+r^*)(X_p - P(1+r)) - M(1+r)$$
$$= M\left[\frac{X_p}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right] \tag{6.1}$$

If $\pi_1 > 0$, the investor should borrow funds domestically, convert domestic funds into foreign funds and invest overseas, and reconvert his foreign currency-denominated amount into his home currency by exercising the put option. If $\pi_1 < 0$, he should probably do the opposite—that is, he should borrow funds from a foreign bank, convert the borrowed funds into home currency at spot market, invest in the home market, and reconvert the home currency-denominated amount by exercising the call option, then take the profit. If the put premium is exactly equal to the call premium, the converse strategy will certainly hold good. Only in the case in which $\pi_1 = 0$ should the investor be indifferent. It is a case of option-based covered interest rate parity, simply defined by the following expression:

$$\frac{X_p}{S}(1+r^*) = (1+r)\left(\frac{P}{S}(1+r^*)+1\right),$$

which can be alternatively shown to be as follows:

$$\left(\frac{X_p - S}{S}\right)(1+r^*) = (r - r^*) + \frac{P}{S}(1+r)(1+r^*) \tag{6.2}$$

Note now that if $P = 0$ and $X_p = F$ (the forward rate of exchange, matching the expiration of the option), then equation 6.2 collapses into the cele-

brated interest rate parity in international finance. Obviously, for $P = 0$ (or, more correctly, for $P > 0$), $X_p > F$.

So far, we have correctly considered the option premium that the investor picks up in the marketplace for his actual transactions. However, theoretically, P is equal to its value, which, *à la* Black and Scholes (1972), is defined by:

$$e^{-rT}P_x\left[1 - N\left(d - \sigma\sqrt{T}\right)\right] - e^{r^*T}S[1 - N(d)],$$

where

$$d = \left\{\left[\ln(S/X_p) + (r - r^* + (\sigma^2/2))T\right]/\sigma\sqrt{T}\right\},$$

σ is the instantaneous standard deviation of the returns on holding of the foreign currency, T is the option's maturity expressed in annualized terms, and $N(\cdot)$ is the standard normal cumulative distribution function. Here if a put-call parity holds good, then

$$P = C + X_p e^{-rT} - Se^{-r^*T}.$$

From this, it is obvious that if $X_p = S$ and/or $T = 0$, $P = C$, and in that event, as noted earlier, $\pi_1 > 0$ means that the investor converts his dollar (*numeraire* currency) into foreign currency and buys put options for the amount generated in the foreign currency, and if $\pi_1 < 0$, he obtains call options as a hedging device. In this exposition further to follow, since multiple rounds of option-covered arbitrage plays are exercised virtually within the time frozen for transactions (in the sense that the market quotes remain unchanged), T, σ, r, r^*, S, X_p, and hence d remains constant, and most likely X_p and S continue to assume the same value. Yet, we must recognize that in any arbitrarily chosen moment, spot-forward parity or put-call parity is not necessarily true, and hence some of these observations should be taken with a grain of salt. Most likely, however, as one may note from real-time data, the parity statements are not empirically observable most of the time, which means that arbitrage opportunities do surface more often than we tend to believe.

As already pointed out, if $\pi_1 \neq 0$, then profit opportunities exist in a risk-free fashion. For the economy of space, let us consider here $\pi_1 > 0$, and let the reader work out the opposite scenario.[3]

$$\pi_1 = M\left[\frac{X_p}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right], \tag{6.3}$$

measures the profit made by the investor that will come to his bank account a month from today, which means that he has earned an amount now equal to $\pi_1/(1 + r) \equiv \pi_{1(0)}$. That is,

$$\pi_{1(0)} = \left(\frac{M}{1+r}\right)\left[\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right]. \tag{6.4}$$

This is the legitimate equity position of this investor right now, immediately after the instant execution of his menu of investment strategies already described, and his banker should not have any problem in recognizing this amount. So, here we assume that in this age of technological speed $\pi_{1(0)}$ is realized and recognized by the transaction-endorsing bank for use by the investor for another round while the market quotes are still unchanged. Since this amount is on hand, the investor seizes the market data to play another round (second round) of arbitrage with his newly created equity $\pi_{1(0)}$. On this second round of play, he makes the amount π_2, which is equal to:

$$\pi_2 = \left(\frac{M}{1+r}\right)\left[\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right]^2,$$

whence:

$$\pi_{2(0)} = \left(\frac{M}{(1+r)^2}\right)\left[\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right]^2$$
$$= \left(\frac{M}{(1+r)}\right)\left[\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right]x$$
$$\left[\frac{\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)}{1+r}\right]^{2-1} \tag{6.5}$$

Similar algebraic manipulations yield then upon the ith iteration:

$$\pi_{i(0)} = \left(\frac{M}{(1+r)}\right)\left[\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right]x$$
$$\left[\frac{\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)}{1+r}\right]^{i-1} \tag{6.6}$$

The summation over all successive iterations ($i = 1, 2, 3, \ldots, n$) results in the following expression:[4]

$$\sum_{i=1}^{n}\pi_{i(0)} = \sum_{i=1}^{n}\left\{\left(\frac{M}{(1+r)}\right)\left[\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right]x\right.$$
$$\left.\left[\frac{\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)}{1+r}\right]^{i-1}\right\}$$
$$= \left(\frac{M}{(1+r)}\right)\left[\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right]\left(\frac{1-\alpha^n}{1-\alpha}\right),$$
$$\text{where } \alpha \equiv \left[\frac{\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)}{1+r}\right]. \tag{6.7}$$

As it is obvious now, the investor can arbitrage iteratively, and make a total profit of π_0^* in n rounds ($n = 1, 2, 3, \ldots$). Here $(1-\alpha^n)/(1-\alpha)$ is the n-round multiplier of the present value of the first round of arbitrage profit, which has already been expressed as follows:

$$\pi_{1(0)} \equiv \left(\frac{M}{1+r}\right)\left[\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right] \tag{6.8}$$

The nth round (as opposed to n-round) arbitrage multiplier is defined, as one can immediately see from equation i, by the expression:

$$\left[\frac{\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)}{1+r}\right]^{n-1}.$$

From these two multipliers, one can note, profit levels may converge to its upper limit:

$$\lim_{n\to\infty}\pi_0^* \equiv \pi_0^{**} = \pi_{1(0)}\left(\frac{1}{1-\alpha}\right).$$

More clearly,

$$\pi_0^{**} = \left(\frac{M}{(1+r)}\right)\left[\frac{X_P}{S}(1+r^*) - (1+r)\left(\frac{P}{S}(1+r^*)+1\right)\right]\left(\frac{1}{1-\alpha}\right), \tag{6.9}$$

if $\alpha < 1$ and $n \to \infty$. If $\alpha > 1$ and $n \to \infty$, profit level asymptotically approaches infinity,—that is, arbitrage profits become unbounded.

Pure Arbitrage Profits with Total Available Funds

Remember in Scenario A, the investor puts back only $\pi_{1(0)}$ into second round of iterative arbitrage with option cover, and in the ith round only the profits made in the $(i-1)th$ round. More rational and profitable strategy in the second round of play, if correctly conceived, should be the one in which the investor uses his strategy with $(M + \pi_{1(0)})$ amount of funds (instead of $\pi_{1(0)}$ amount), which then churns into the following level of profits in the second round:

$$\hat{\pi}_{2(0)} = \left(\frac{1}{1+r}\right)[(M+\pi_{1(0)})/S]\left\{\frac{M}{S}(1+r^*)(X_P - P(1+r))\right\}$$
$$-(M+\pi_{1(0)})(1+r) = M\alpha(1+\alpha), \tag{6.10}$$

and on the ith round, the profit level amounts to:

$$\hat{\pi}_{i(0)} = \left(\frac{1}{1+r}\right)[(M+\pi_{1(0)}+\pi_{2(0)}+\ldots\pi_{i-1(0)})/S].$$
$$\left\{\frac{M}{S}(1+r^*)(X_P - P(1+r))\right\} - (M+\pi_{1(0)}+\pi_{2(0)}+\ldots\pi_{i-1(0)}))(1+r)$$
$$= M\alpha(1+\alpha)^{i-1} \tag{6.11}$$

The summation over all the successive iterations ($i = 1, 2, 3, \ldots, n$) results in the following expression in this modified situation:

$$\hat{\pi}_0^* = \sum_{i=1}^{n}\hat{\pi}_{i(0)} = \sum_{i=1}^{n} M\alpha(1+\alpha)^{i-1} = M\alpha\left[\frac{1-(1-\alpha)^n}{1-(1+\alpha)}\right]. \tag{6.12}$$

Since $\alpha > 0$, $(1+\alpha) > 1$, and hence $\hat{\pi}_0^{**}(\equiv \lim_{n\to\infty}\hat{\pi}_0^*)$ is unbounded. In simple terms, the investors can generate unlimited amount of profits by fully exploiting market misalignment.

Arbitrage Profits with Total Available Funds: True Measure

In the earlier section dealing with pure arbitrage with total available funds, we have measured profit level in round 2 as follows:

$$\hat{\pi}_{2(0)} = \left(\frac{1}{1+r}\right)[(M+\pi_{1(0)})/S]\left\{\frac{M}{S}(1+r^*)(X_P - P(1+r))\right\} - (M+\pi_{1(0)})(1+r).$$

Note that since $\pi_{1(0)}$ is the investor's money just made, he may not feel like including interest cost on this amount in the calculation of capital costs to

be subtracted in profit computation—that is, his profit measure in second round should *truly* be equal to $(\tilde{\pi}_{2(0)})$:

$$\tilde{\pi}_{2(0)} = \left(\frac{1}{1+r}\right)[(M+\pi_{1(0)})/S]\left\{\frac{M}{S}(1+r^*)(X_P - P(1+r))\right\} - M(1+r)$$
$$= M\alpha^2 \tag{6.13}$$

and similarly, on his ith round, profits must be measured by:

$$\tilde{\pi}_{i(0)} = \left(\frac{1}{1+r}\right)[(M+\pi_{1(0)}+\pi_{2(0)}+\ldots\pi_{i-1(0)})/S].$$
$$\left\{\frac{M}{S}(1+r^*)(X_P - P(1+r))\right\} - M(1+r) = M\alpha^i \tag{6.14}$$

The cumulative profits for n rounds then must be as follows:

$$\pi_0^* = \sum_{i=1}^{n}\tilde{\pi}_{i(0)} = M\alpha\left(\frac{1-\alpha^n}{1-\alpha}\right) \tag{6.15}$$

OPTIONS COVERED CURRENCY ARBITRAGE WITH TRANSACTION COSTS

Here we introduce transaction costs exactly in the way these costs appear in a trading structure. Let us bring out then *ask* and *bid* quotes in the exchange rates, and *deposit* and *borrowing* rates of interest in home and foreign markets as follows:

S^A = *ask* spot rate of exchange (of, say, 1 French franc in terms of U.S. dollar(s));
S^B = *bid* spot rate of exchange (of, say, 1 French franc in terms of U.S. dollar(s));
$X^T{}_P$ = exercise price of 1-month put option inclusive of brokerage fee;
P^T = put premium inclusive of brokerage fee, which is usually equal to $P(1 + t)$, where t measures the *ad valorem* fee on transaction;
$X^T{}_C$ = exercise price of a one-month call option inclusive of brokerage fee;
C^T = call premium inclusive of brokerage fee, which is usually equal to $C(1 + t)$, where t measures the *ad valorem* fee on transaction;
r_D = (domestic *deposit*) interest rate for one month;
r_B = (domestic *borrowing*) interest rate for one month;
r^*_D = (foreign *deposit*) interest rate for one month;
r^*_B = (foreign *borrowing*) interest rate for one month.

If the investor borrows M dollars and performs the same operations he does in the section dealing with pure arbitrage profits with newly created profits, his first round profits should be measured by the following expression:

$$\pi_1^T = M\left[\frac{X_P^T}{S^B}(1+r_D)-(1+r_B)\left(\frac{P^T}{S^B}(1+r_D^{*})+1\right)\right], \tag{6.16}$$

where π^T_1 denotes arbitrage profits in the first round with transaction costs. $\pi^T_1/(1 + r_B) \equiv \pi^T_{1(0)}$ is the present value of the first round of arbitrage profits under transaction costs. Now, following the same procedure as in the earlier section, one can get the following expression for the ith round of arbitrage profits under transaction costs:

$$\pi_{i(0)}^T = \left(\frac{M}{(1+r_B)^i}\right)\left[\frac{X_P^T}{S^B}(1+r_L^{*})-(1+r_B)(1+r_L^{*})\left(\frac{P^T}{S^B}\right)\right]^i$$

$$= \left(\frac{M}{(1+r_B)}\right)\left[\frac{X_P^T}{S^B}(1+r_L^{*})-(1+r_B)\left(\frac{P^T}{S^B}(1+r^{*})+1\right)\right]\times$$

$$\left[\frac{\frac{X_P^T}{S^B}(1+r_L^{*})-(1+r_B)\left(\frac{P^T}{S^B}(1+r^{*})+1\right)}{1+r}\right]^{i-1},$$

and hence,

$$\pi_0^{T*} =$$

$$\sum_{i=1}^{n}\pi_{i(0)}^{T*} = \left(\frac{M}{1+r_B}\right)\cdot\left\{\left(\frac{M}{(1+r_B)}\right)\left[\frac{X_P^T}{S}(1+r_D^{*})-(1+r_B)\left(\frac{P^T}{S^B}(1+r_D^{*})+1\right)\right]\left(\frac{1-(\alpha^T)^n}{1-\alpha^T}\right)\right\},$$

where $\alpha^T \equiv (X_P^T/S^B)(1+r_D^*)-(1+r_B)((P^T/S^B)+1)$. The derivations with transaction costs in Scenario B and in Scenario C are straightforward, and since it will simply consume too much space and give rise to some tedium, although the final forms are very significant for the investor, we leave those chores for the interested readers.

It is not difficult to realize that profits at each iteration under transaction costs are reduced, and collective rounds yield reduced levels of profits as a result. It also becomes quite comprehensible that in some instances profits may completely disappear or even turn into losses.

MICROSTATICS AND MACRODYNAMICS OF THE MARKET: SOME OBSERVATIONS

It is a fundamental reality that if arbitrage opportunity exists in the marketplace, it soon disappears by the dynamics of competition, and in that sense it may appear quite questionable if the second, and third, and the nth round of arbitrage activities of our investor can ever take place. A careful reflection of this point is absolutely essential, and upon that reflection and comprehension of market forces, one should realize the following points. First of all, if the investor finds that arbitrage opportunity exists, say, by the satisfaction of the condition, given by equation 6.1, he ascertains profits *instantly* for all the n rounds of arbitrage. The market data are the same for round 1 and round n within 20 seconds or 2 minutes in which quotes do not change from the investor's screen. If his first finger presses a key on his first computer, his 10th finger presses the same programmed key on his 10th computer *almost* at the very same instant. The first and the 10th round differ, however, only by the amounts of arbitrage funds—in the first round the amount is M dollars, and on the nth round the amount is $\pi_n/(1+r)^n$ and the time involved in these 10 successive rounds may be less than a second with digitized signatures of approval by the bank(s) in the middle with today's technology and speed. The moment the market data are factored in, and π_1 (? 0) is ascertained, one computes $\pi_{1(0)}$, $\pi_{2(0)}$, $\pi_{3(0)}$, …, $\pi_{n(0)}$, and so on. If the investor can exploit the market one time via arbitrage, he can exploit the same market several times, since the moment is *virtually* frozen, and the data for market exploitation remain the same. Note that the investor is a micro agent operating in the marketplace in which even the speediest adjustment cannot deprive him of the opportunity to take advantage of the market misalignment. We know for sure that arbitrage exists in the market, and many players subsist on it. Therefore, arbitrage and iterations are valid plays in the market. One should also note that in the trillion-dollar market, a million or even a few billions by a micro agent may not throw the market into any state of concussion. However, if a large number of participants act in the same moment, there may be an execution jam, and nobody is likely to make any profits out of arbitrage. In the situation of multiple players, the macrodynamics of the market set in and force arbitragers into a zero-profit condition. One more point should be made. Since too many iterations are involved in this investment strategy, one should realize that before some iteration is executed, some quotes may change. So in order to guard against this possibility, an appropriate limit order should be put in with each iteration of covered arbitrage, and it should guarantee nonnegative profit conditions in the repeated arbitrage acts.

Market dynamics and market efficiency are of paramount significance. Cornell (1977) examines these concerns quite efficiently. For further examination of our analytical framework, let us consider the adjustment process in the dynamic macrostructure of the market, which can be captured in the following way:

$$d\omega/dt = \lambda(\omega - \omega^{*}),$$

where $\omega/X_p/S$, ω^{*} is the value of in which option-based covered interest rate parity, as defined earlier by equation (6.2), holds, and $\lambda > 0$ is the speed of adjustment. The solution to this differential equation is given by:

$$\omega(t) = \omega^{*} + (\omega_0 - \omega^{*})e^{-\lambda t}$$

Here ω_0 is the initial value of X_p/S. As $t \rightarrow \infty$, $\omega \rightarrow \omega^{*}$. One should note then that in the frozen (static) moment, no adjustment is possible, and micro-level arbitrage is an exploitable opportunity.

At this point, a few more issues should be addressed as well. One may wonder why an investor who sees an arbitrage opportunity will start off with $1 million instead of hundreds of millions or billions of dollars. The answer is simple. If the investor has $1 million as the maximum amount available to him, he only has to begin with that much money. *M* dollars, in our paradigm, is the maximum available initial fund. Of course, if the investor has more, he will initiate his moves with more funds. The issue here is not what the optimal amount of initial investment funds for arbitrage should be; the issue is: if an initial amount—be it *M* dollars or *Z* dollars—is available for arbitrage, what amount of money can potentially be generated out of that initial situation? Two other issues should be brought to light in this context. One may argue that since profits out of the first round of arbitrage are obtained only at the end of one month from that day, how is this investor getting funds for second, and third, and other rounds of market plays? Note here that π_1 is a sure amount of money made by the investor without taking any risk, and any bank should recognize this amount the investor makes at the end of one month. If this is a common knowledge of the investor as well as that of the bank, it is equally recognizable that this investor has $\pi_{1(0)}/\pi_1/1 + r)$ now—and it is his equity position, which he can legitimately utilize (probably with a prior discussion with his banker). It is worth noting, particularly against the backdrop of the common belief that markets are so well-aligned that scope for arbitrage in reality is nonexistent, that in the currency market one can almost always find arbitrage opportunity. Note that although spot rate and option prices are *usually* defined at a point of time, and corresponding to those defined quotes a set of domestic and foreign interest rates will yield $\pi_1 = 0$, one can always find another set of interest rates from the available spectrum of interest rates, which generates $\pi_1 \geqslant 0$. This clearly signifies that arbitrage opportunity is a viable and feasible strategy in the foreign exchange market more often than not. Additionally, it should be pointed out that one who watches real-time data can easily recognize that the quotations on spot rate and options prices by different banks and/or dealers are not always the same at the same instant. So on that front one may also find the scope for arbitrage. Finally, we should note that if one round of arbitrage act is undertaken, it may appear that arbitrage profit is negligible, and in that sense one may conclude that arbitrage opportunity is virtually nonexistent.

NOTES

1. See other citations in the references at the end.

2. Here r and r^{*} are the interest rates for one month—not the annual interest rates. If one has annual interest rates available, the investor should divide those rates by 12 to get to the (acceptable) one-month interest rates matching, say, an option maturing in a month. Since one-month options are being considered here, we have chosen one-month compounding/discounting in this work. But, technically one should note that the investor can choose any compounding or discounting interval, even continuous compounding or discounting for computations, and profit measures as a result will get modified. The author would like to acknowledge his indebtedness to the participants in the seminar in Marseille who suggested this point.

3. It should be clearly noted at this point that if equation (6.1) assumes negative value, it does not automatically signify positive profit condition under investment strategy in which the investor borrows funds from a foreign country and starts from the other end. It is instructive, therefore, that we define profit measures under different arbitrage plays when the investor is borrowing, say, *N* French francs at r^{*}, converting the amount into U.S. dollars at the spot rate of exchange (S), investing the converted dollar amount in the U.S. market at r, exercising call option and buying French francs, and finally paying off the original debt and the accrued interest on that debt. In this case, his profits in the first round of arbitrage ${}^{f}\pi_1$ are computed as follows:

$$\begin{aligned}{}^{f}\pi_1 &= NS(1 + r)\{(1/X_C) - (1/C)(1 + r^{*})\} - N(1 + r^{*}) \\ &= N[(S/X^{C})(1 + r) - (1 + r^{*})\{(S/C)(1 + r) + 1\}], \end{aligned} \qquad (6.17)$$

and therefore, the present value of the amount is:

$${}^{f}\pi_{1(0)} = (N/(1 + r))[(S/X^{C})(1 + r) - (1 + r^{*})\{(S/C)(1 + r) + 1\}], \qquad (6.18)$$

and on the ith iteration, then profits are defined by:

$$\begin{aligned}{}^{f}\pi_{i(0)} &= (N/(1 + r^{*})^{i})[[(S/X^{C})(1 + r) - (1 + r^{*})(1 + r)(S/C)]^{i} \\ &- [(S/X^{C})(1 + r) - (1 + r^{*})(1 + r)(S/C)]^{i-1}(1 + r^{*})]. \end{aligned} \qquad (6.19)$$

Note here that (1) and (^{f}1), and, in more general situations, (i) and (^{f}i) are not *necessarily* of opposite signs.

4. It should be noted, as the reviewer pointed out aptly, that there may be an institutional ceiling on the amount an investor can go after with these iterations. A

creative design may be in order to circumvent such institutional restriction if it exists.

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