Chapter 5

Arbitrage and Hedging with Spot and Forward Contracts

INTRODUCTION

Arbitrage, as noted in chapter 1, is the exploitation of market misalignment, and hedging is the cover against any open risk-exposed positions of a participant in the marketplace. Academic research has taken us to two interesting ends on this issue of arbitrage with hedging. At one end, we find that theoretical explorations of market potential have opened the eyes of actual traders and made them realize the fruits of the mechanics of financial markets. Asset markets in general—and currency market in particular—have created conditions in which players make money without even taking any risk through arbitrage, and on many occasions they generate large amounts of profits by taking speculative positions—sometimes covered and sometimes naked.[1] On the other end of the spectrum, we note that it is the academic maxim mostly that intrigues others by letting them believe that markets are so well-aligned that arbitrage opportunity can hardly exist in the real world, and hence risk-free profit-taking is a mere illusion. In this chapter, an attempt is made to examine how correct that analytical view is against the setting of real markets, how much academic research helps us understand the behavior of real traders, and to what extent the impression or belief that there is no scope for arbitrage holds ground. We shall attempt to check into the foreign exchange market in which currencies are traded for spot and forward contracts almost 24 hours a day, where traders hardly get out of trading rooms. The plan of this chapter is as follows: In the following next five sections under the heading "Arbitrage Profits," we bring out the conditions for profitable arbitrage with full hedging with and without transaction costs, and ascertain the *minimum* and *maximum* (possible) gains (profits) out of trading acts under admissible situations. The theoretical designs of operational schemes are discussed under different possible scenarios with and without leveraged market moves. Each scenario is then tested with real-time data, taken mostly from *Reuters*, and rechecked with bankers and dealers. In the section under "Arbitrage and Leverage," arbitrage with leverage and arbitrage-induced total profits are measured when the trader operates with transaction costs. The next section reexamines the arbitrage profits without transaction costs once again. The following sections discuss covered interest triangular arbitrage profits, followed by observations on micro-structure and dynamics of market competition. Some empirical evidence, taken from the *Reuters* real-time data screen, is examined in the context of the scope for arbitrage, and profit measures and profit multipliers are computed. These results are then presented in Table 5.1. Finally, in the last section, we conclude the work with some observations. Many comments in regard to the speed of transaction, logistics, and the feasibility of the operational success, execution jam, limit order, and so on are made at appropriate points in our discussion.

ARBITRAGE PROFITS: LOWER AND UPPER BOUNDS

In currency markets throughout the world, active participants trade around the clock, going long and/or short often enough and making foreign exchange trading the most voluminous structure of asset-market transactions. This market has various facets, and here we plan to take up the trading in spot and forward contracts along with participation in money market. A significant research along the line has been done by Keynes (1923), Aliber (1973), Frenkel and Levich (1975), and Deardorff (1979). More recently, Blenman (1992a, 1992b, 1996), Blenman and Thatcher (1995), Rhee and Chang (1992), Callier (1981), Clinton (1988), Ghosh (1991, 1994, 1997), McCormick (1979), Roll and Solnik (1979), Tsiang (1959, 1973), and a few others have extended the discussions somewhat further.[2] Our approach here is to take stock of the research to date in some measure, then attempt to assess the degree to which we have been or have not been able to exploit the market to its full potential by our financial engineering.

Arbitrage Profits without Transactions Costs

Assume that transaction costs are so low that one can virtually ignore them as nonexistent. Later in this chapter, we discard this simplifying assumption, and enter into the world in which transactions costs are significant. The presence of transaction costs is introduced in the way they appear in trade transactions. For now, let S be the current spot rate of

exchange of, say, one French franc in terms of U.S. dollar(s), F the currently traded forward rate with T-day maturity (T = 30, 60, 90, 180, 360, etc.), r and r^*, the U.S. and French interest rates, respectively, for the period matching the forward contract we are considering here.[3]

When an investor has the quotations of these exchange rates and interest rates either from her computer screen in real time or from her broker or from bank telephone calls, she has, for our examination, basically two alternative investment strategies: (i) borrow, for instance, M dollars in the U.S. market at the rate r, convert the borrowed amount in French francs at the spot rate of exchange S, invest the converted French franc amount (M/S) at r^* and get the following amount $(M/S)(1 + r^*)$ at the end of T days; (ii) borrow the equivalent amount of French francs, say N French francs at r^*, convert into M dollars (that is, $M = N{\cdot}S$), invest these M dollars at the rate r, and get $M(1 + r)$ dollars at the end of T days. But that is not all. Under alternative (i), the investor sells $(M/S)(1 + r^*)$ francs at the forward rate at the very moment she buys francs in the spot market, and thus turns her initial investment dollars into the amount of $(M/S)(1 + r^*)F$ without entering into risk anywhere in the process. At this stage, she must subtract $M(1 + r)$ from $(M/S)(1 + r^*)F$, and compute her T-day profits (π_1) in this play of covered arbitrage as follows:

$$\pi_1 = \frac{M}{S}(1+r^*)F - M(1+r) = M\{(\frac{F}{S})(1+r^*) - (1+r)\}. \tag{5.1}$$

If $\pi_1 > 0$, the investment strategy (i) is profitable. By going through the similar algebraic steps, one may easily see that $\pi_1 < 0$ (which yields loss under strategy (i)) signifies the profitable condition under alternative (ii), as outlined above. Obviously, $\pi_1 = 0$ means the zero-profit situation under the given parametric environment.

It is now instructive that we explore further into the conditions in which $\pi_1 = 0$. For economy of space, we deal with $\pi_1 > 0$, and the sign reversal (if that happens to be the case) should simply be construed as taking the other alternative investment strategy for positive profit condition.

Profit Multiplier in the Absence of Transaction Costs

It has been already shown that if, for instance, $\pi_1 > 0$ holds, then the investor makes the following amount of total profits at the end of T days:

$$\pi_1 = M\{(\frac{F}{S})(1+r^*) - (1+r)\}. \tag{5.2}$$

The present value of this profit π_1 is then as follows:

$$\pi_1^0 = \frac{M}{(1+r)}\{(\frac{F}{S})(1+r^*) - (1+r)\}. \tag{5.3}$$

This is the measure of the *minimum* bound of the investor's profits now in the risk-free activities in the currency market under the given situation. Since this is the amount the investor owns *now*, she should make use of this amount instantly to play the market still with the same quotations in the exchange and money markets. Here is then the second round of profits:

$$\pi_2 = ((\frac{\pi_1^0}{S})(1+r^*)F - \pi_1^0(1+r) = \pi_1^0\{(\frac{F}{S})(1+r^*)-(1+r)\},$$

which is readily reduced to the expression:

$$\pi_2 = [\frac{M}{(1+r)}]\{(\frac{F}{S})(1+r^*)-(1+r)\}^2,$$

the present value of which then is:

$$\pi_2^0 = [\frac{M}{(1+r)^2}]\{(\frac{F}{S})(1+r^*)-(1+r)\}^2. \tag{5.4}$$

The present value of profits upon *i*th iteration ($i = 1, 2, \ldots, n$) is then:

$$\pi_i^0 = [\frac{M}{(1+r)^i}]\{(\frac{F}{S})(1+r^*)-(1+r)\}^i,$$

$$= \left(\frac{M}{1+r}\right)\{(\frac{F}{S})(1+r^*)-(1+r)\}.[\frac{1}{(1+r)^{i-1}}\{(\frac{F}{S})(1+r^*)-(1+r)\}^{i-1}]. \tag{5.5}$$

Let $\mu(i) \equiv [1/(1+r)^{i-1}\{(F/S)(1+r^*)-(1+r)\}^{i-1}]$ be the arbitrage profit multiplier of the initial profits at the *i*th round of arbitrage ($i = 1, 2, 3, \ldots n, \ldots, 4$). From equation 5.5, one can get for n successive iterations of covered arbitrage:

$$\pi_{(16n)}{}^0 \equiv \sum_{i=1}^{n} \pi_i^0 = M \sum_{i=1}^{n} (\frac{1}{(1+r)^i})\{(\frac{F}{S})(1+r^*)-(1+r)\}^i$$

$$= M\alpha\beta\left[\frac{1-(\alpha\beta)^n}{1-\alpha\beta}\right] \tag{5.6}$$

$$\text{where } \alpha \equiv \frac{1}{(1+r)} \text{ and } \beta/\{(\frac{F}{S})(1+r^*)-(1+r)\}$$

$$\text{For } n = \infty,\ \pi_{(164)}{}^0 = M\alpha\beta\left[\frac{1}{1-\alpha\beta}\right] \text{ (if } \alpha\beta < 1\text{); otherwise } \pi_{(164)}{}^0 = \infty. \tag{5.7}$$

Equation 5.7 defines the *maximum* bound of profits the investor can make out of the given situation. One can see now that $M\alpha\beta$ =

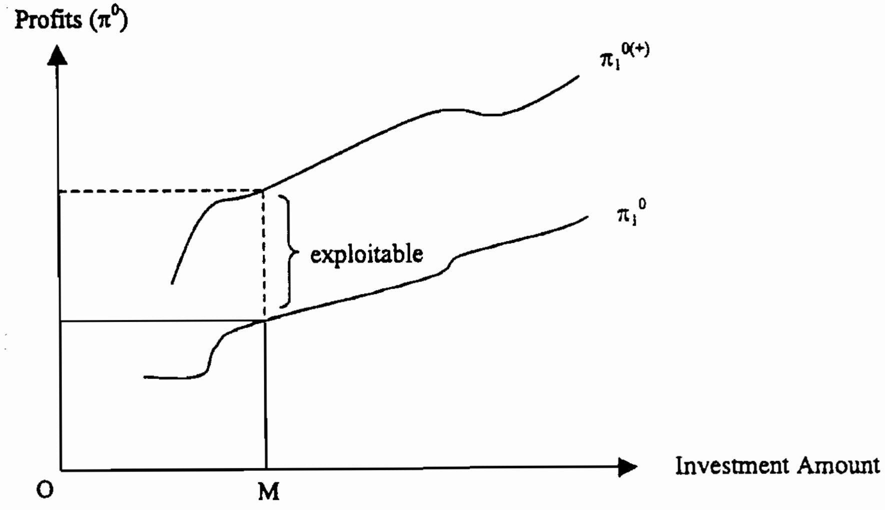

Figure 5.1 Investment and profits on single and cumulative iterations

her arbitrage profits, and $\mu^+(n)/\{1-(\alpha\beta)^n\}/\{1-\alpha\beta\}$ is the profit multiplier for *n successive* rounds of (arbitrage) profits. Figure 5.1 portrays the boundaries.

In Figure 5.1, the horizontal axis measures the initial investment funds of the investor, and the vertical axis measure the present value of cumulative profits for first *i* rounds of arbitrage ($i = 1, 2, 3, \ldots, n, \ldots, \infty$). If the initial amount invested is *M* dollars, then upon first round of arbitrage, the investor makes a net profit of ${}_M\pi_1{}^0$ now, and on first n rounds of arbitraging she makes the amount of ${}_M\pi_n{}^{(+)0}$. The rays $\pi_1{}^0$ and $\pi_n{}^{(+)0}$ in Figure 5.1 define the relationship between initial investment amount and total profit levels at different cumulative arbitrage rounds. As Figure 5.1 shows, if the initial amount instead were Z (> *M*), then those profit measures would be different. At this point, a point should be noted: if the investor does not go beyond the first round of arbitrage with initial amount, say, *M* dollars, her unexploited profit is the difference between ${}_M\pi_n{}^{(+)0}$ and ${}_M\pi_1{}^0$.

To comprehend the significance of our analysis, let us examine the amount of profits (a rational trader can generate) and the magnification thereof upon different iteration of covered arbitrage operations, and on that score, consider the following data taken out of *The Wall Street Journal* and *Financial Times* as an illustration: M = \$1,000,000, S = 0.2022, F = 0.2035, r = 0.0447, and r^* = 0.0445, T = 360 days, and θ = 0. With this market data, the arbitrageur makes a profit of \$4,612.91 on her first round, \$5,816.88 on her 25th round: but for 25 successive rounds, she earns a net round of arbitrage profits, but any rational investor can hardly ignore the cumulative amount at a given instant of time. If the investor can make it to the round 100 in this illustrative case, then her cumulative profits amount to \$773,505.90. When we admit 25 percent borrowing against the equity position (that is, $\theta = 25\%$), one can find that the computed values of profits—at individual round and at cumulative rounds—change (increase), but not very significantly until higher levels of iterations are made.

Arbitrage-Induced Total Profits without Transactions Costs

Thus far, we have dealt with the repeat exploitations of the foreign exchange market admitting of arbitrage opportunities, and arbitrage profits at each round of play have been computed. But note that in the first round the investor has played the borrowed funds either from a bank or from herself at the going market rate r (which must be construed as the opportunity cost for her own money in the event the initial amount was taken out of her own purse), and hence $M(1 + r)$ must be subtracted from the money made in the first round. But from second round onward, she should *stop* deducting interest expenses. What it all means is that, from second round on, profit calculations should be as follows:

$$\pi^{+}_{T(2)} = \left(\frac{\pi^{0}_{1}}{S}\right)(1+r^{*})F,$$

and therefore,

$$\pi^{(+)0}_{T(2)} = \frac{\frac{\pi^{0}_{1}}{S}(1+r^{*})F}{(1+r)}$$
$$= \left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)(1+r^{*})-(1+r)\right\}\left\{\frac{1}{(1+r)^{2}}\right\}\left[\left(\frac{F}{S}\right)(1+r^{*})\right], \quad (5.8)$$

and similarly,

$$\pi^{(+)0}_{T(3)} = \left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)(1+r^{*})-(1+r)\right\}\left\{\frac{1}{(1+r)^{2}}\right\}\left[\left(\frac{F}{S}\right)(1+r^{*})\right]^{2} \quad (5.9)$$

and

$$\pi^{(+)0}_{T(i)} = \left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)(1+r^{*})-(1+r)\right\}\left\{\frac{1}{(1+r)^{i-1}}\right\}\left[\left(\frac{F}{S}\right)(1+r^{*})\right]^{i-1} \quad (5.10)$$

Here the superscript + in π denotes the cumulative magnitude of profits, superscript 0 refers to the present value of the profit, and subscript $T(i)$ refers to the ith iteration of arbitrage in the calculation of total arbitrage-induced profits as opposed to pure total arbitrage profits ($i = 1, 2, 3, \ldots n, \ldots, 4$). The summation over first n rounds then yields:

$$\pi^{(+)0}_{T(1\to n)} \equiv \sum_{i=1}^{n} \pi^{(+)0}_{T(i)}$$
$$\equiv \sum_{i=1}^{n}\left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)\times(1+r^{*})-(1+r)\right\}\left\{\frac{1}{(1+r)^{i-1}}\right\}\left[\left(\frac{F}{S}\right)(1+r^{*})\right]^{i-1}, \quad (5.11)$$

$$= \left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)(1+r^{*})-(1+r)\right\}\left[\frac{\left[1-\left(\frac{F}{S}\frac{1+r^{*}}{1+r}\right)^{n}\right.}{1-\left(\frac{F}{S}\frac{1+r^{*}}{1+r}\right)}\right] \quad (5.12)$$

and, for $n = \infty$, if $\frac{F}{S}\left(\frac{1+r^{*}}{1+r^{*}}\right) < 1$

$$\pi^{(+)0}_{(1\to\infty)} \equiv \left(\frac{M}{1+r}\right)\left[\frac{F}{S}(1+r^{*})-(1+r)\right]\times\left(\frac{1}{1-\left(\frac{F}{S}\frac{1+r^{*}}{1+r}\right)}\right) \quad (5.13)$$

When $\frac{F}{S}\left(\frac{1+r^{*}}{1+r}\right) > 1$ infinite iterations result in infinite amount of profits. So the bounds for profits—lower and upper—are defined as follows:

$$\pi^{0}_{T(1)} = \left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)(1+r^{*})-(1+r)\right\}: \quad \text{lower bound;}$$

$$\pi^{(+)0}_{T(1\to\infty)} = \left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)(1+r^{*})-(1+r)\right\}\left[\frac{1}{1-\left(\frac{F}{S}\frac{1+r^{*}}{1+r}\right)}\right]: \quad \text{upper bound,}$$

$$\left(\text{when } \left(\frac{F}{S}\right)\frac{(1+r^{*})}{(1+r)} < 1\right),$$

$$\pi^{(+)0}{}_{T(1\to\infty)} = \infty \qquad \left(\text{when } \left(\frac{F}{S}\right)\frac{(1+r^{*})}{(1+r)} > 1\right).$$

One may note that $\mu^+(n) \equiv \{1 - (\alpha\beta)^n\}/\{1 - \alpha\beta\}$ is the *(pure arbitrage) profit multiplier*, and $\mu^+_T(n) = [1 - \{(F/S)(1 + r^*)/(1 + r)\}^n]/\{1 - (F/S)(1 + r^*)/(1 + r)\}$ is the *arbitrage-induced profit multiplier* for n rounds ($1 \leq n \leq 4$). Similarly, $\pi_{(1\to\infty)}{}^{(+)0}$ and $\pi_{T(1\to\infty)}{}^{(+)0}$ are the *pure* arbitrage and *arbitrage-induced* total profits for the first n rounds.

Arbitrage Profits with Transactions Costs

Now, ease up the assumption that transactions costs are nonexistent, and introduce those costs indeed in the way they factor in—that is, by way of their appearing in the form of *bid* (buy) and *ask* (sell) quotations in foreign exchange rates. Let S^B and S^A be the spot *bid* and *ask* prices (of dealers or banks) of, say, 1 French franc in terms of U.S. dollars, F^B and F^A be the T-day forward *bid* and *ask* quotations. Next, denote the bank's deposit rate (or its borrowing rate—the investor's earnings rate on her deposits) and lending rate (which is the investor's borrowing rate) in the United States by r_D and r_L, and $r_D{}^*$ and $r_L{}^*$ in France, respectively. With these notations in place, the investor makes the following amount of profits when she enters into the investment strategy (i), as outlined earlier:

$$\pi_{t(1)} = \frac{M}{S^A}(1+r_D^*)F^B - M(1+r_L)$$

$$= M\{\frac{F^B}{S^A}(1+r_D^*) - (1+r_L)\} \tag{5.14}$$

whence the present value is:

$$\pi^0_{t(1)} = \left(\frac{M}{1+r_L}\right)\{\frac{F^B}{S^A}(1+r_D^*) - (1+r_L)\}. \tag{5.15}$$

Here $\pi_{t(1)}$ stands for profits for the first round of arbitrage with transaction costs, and $\pi^0{}_{t(1)}$ is the present value of first-round arbitrage profits. Under transactions costs this is the *minimum* amount of arbitrage profits for the investor. The present value of the ith iteration of the market play with the same data is then measured by the following:

$$\pi^0_{t(1)} = \left(\frac{M}{1+r_L}\right)\{\frac{F^B}{S^A}(1+r_D^*) - (1+r_L)\}^i, \tag{5.16}$$

and hence the sum total of the present value of profits from round 1 through n is:

$$\pi^{(+)0}_{t(1\to n)} \equiv M\sum_{i=1}^{n}\frac{1}{(1+r_L)^i}\{\frac{F^B}{S^A}(1+r_D^*) - (1+r_L)\}^i = M\alpha_t\beta_t\left(\frac{1-(\alpha_t\beta_t)^n}{1-\alpha_t\beta_t}\right), \tag{5.17}$$

where $\alpha_t \equiv 1/(1 + r_L)$ and $\beta_t \equiv (F^B/S^A)(1 + r^*_D) - (1 + r_L)$. As before, for $n = \infty$, and $\alpha_t\beta_t < 1$,

$$\pi^{(+)0}_{t(1\infty)} = M\alpha_t\beta_t\left(\frac{1}{1-\alpha_t\beta_t}\right), \text{ and for } \alpha_t\beta_t > 1,\ \pi^{(+)0}_{t(1\infty)} = \infty. \tag{5.18}$$

Note that subscript t signifies the presence of transaction costs. Here $M\alpha_t\beta_t$ is the present value of the first round of riskless profits in currency markets, and $1/(1 - \alpha_t\beta_t)$ is the multiplier for $n = \infty$ (if $\alpha_t\beta_t < 1$). This is the *maximum* amount of profits attainable under transactions costs under investment strategy (i).[4]

Following the procedure outlined in an earlier section, one can easily get the following expressions of arbitrage-induced total profits for the ith iteration and the cumulative value of arbitrage-induced total profits up to and inclusive of the ith iteration in the presence of transaction costs:

$$\pi^{(+)0}_{Tt(i)} = \left(\frac{M}{1+r_L}\right)\left\{\frac{F^B}{S^A}(1+r_D^*) - (1+r_L)\right\}\cdot\left(\frac{F^B}{S^A}\frac{1+r_D^*}{1+r_L}\right)^{i-1} \tag{5.19}$$

$$\pi^{(+)0}{}_{Tt(1\to n)} \equiv \sum_{i=1}^{n}\pi^{(+)0}{}_{Tt(i)}\left(\frac{M}{1+r_L}\right)$$

$$\equiv \sum_{i=1}^{n}\{\frac{F^B}{S^A}(1+r^*{}_D) - (1+r_L)\}\left(\frac{F^B}{S^A}\frac{1+r_D^*}{1+r_L}\right)^{i-1}. \tag{5.20}$$

So, the bounds for profits—lower and upper—are defined in this instance (under transactions costs) as follows:

$$\pi^0_{Tt(1)} = \frac{M}{(1+r_L)}\left\{\left(\frac{F^B}{S^A}\right)(1+r_D^*) - (1+r_L)\right\}: \quad \text{lower bound;}$$

$$\pi^{(+)0}_{Tt(1\to n)} = \left(\frac{M}{1+r_L}\right)\left\{\left(\frac{F^B}{S^A}\right)(1+r_D^*) - (1+r_L)\right\}\left[\frac{1-\left(\frac{F^B}{S^A}\frac{1+r_D^*}{1+r_L}\right)^n}{1-\left(\frac{F^B}{S^A}\frac{1+r_D^*}{1+r_L}\right)}\right]: \quad \text{upper bound.}$$

When $n = \infty$, the upper bound is equal to:

$$\pi^{(+)0}_{Tt(1\to\infty)} = \left(\frac{M}{1+r_L}\right)\left\{\left(\frac{F^B}{S^A}\right)(1+r^*_D)-(1+r_L)\right\}\left[\frac{1}{1-\left(\frac{F^B}{S^A}\frac{1+r^*_D}{1+r_L}\right)}\right] \text{: upper bound}$$

$$\text{(when } (F^B/S^A)(1+r^{*D})/(1+r_L)<1\text{)},$$

$$\pi^{(+)0}_{Tt(1\to\infty)} = \infty \qquad \text{(when } (F^B/S^A)(1+r^{*D})/(1+r_L)>1\text{)}.$$

ARBITRAGE WITH LEVERAGE: TOTAL (ARBITRAGE-INDUCED) PROFITS WITH TRANSACTIONS COSTS

Leverage refers to the extent of debt the investor uses in her total investable funds for arbitrage in the foreign exchange market. In this analytical structure, borrowed funds have indeed been introduced as the initial investment dollars (or francs) for the investor. Now, consider the scenario in which leverage is further utilized by our investor at every step of her market moves. It is assumed now that the investor has the ability to borrow θ percent of every dollar amount of equity (or its equivalent amount in another currency) she has at the going market rate r_L (or at r^*_L in case of the other currency). This additional feature then requires the necessary modifications in the previous derivations, and upon those modifications, one must get the following present value of profits at the first round:

$$\pi^0_{Tt(1)} = \left(\frac{M}{1+r_L}\right)\left\{\frac{F^B}{S^A}(1+r^*_D)-(1+r_L)\right\}, \tag{5.21}$$

which is exactly the same value as (5.15). Since this much profit is the present value of the investor's total (pure and arbitrage-induced) profits, she can borrow $\theta\pi^0_{Tt(1)}$ from her bank, and thus put $\pi^0_{Tt(1)}(1 + \theta)$ in arbitrage process to generate the following amounts of arbitrage-induced total profits in the next round (second round in this instance):

$$\left(\frac{\pi^0_{Tt(2)}(1+\theta)}{S^A}\right)(1+r^*_D)F^B - \pi^0_{Tt(1)}\theta(1+r_L),$$

the present value of which is then:

$$\pi^0_{Tt(2)} = \left(\frac{M}{1+r_L}\right)\left\{\frac{F^B}{S^A}(1+r^*_D)-(1+r_L)\right\}\cdot \left[\left(\frac{1}{1+r_L}\right)\left\{\frac{F^B}{S^A}(1+r^*_D)+\theta\left\{\frac{F^B}{S^A}(1+r^*_D)-(1+r_L)\right\}\right\}\right] \tag{5.22}$$

and then, for round i ($i = 1, 2, \ldots, n$),

$$\pi^0_{Tt(i)} = \left(\frac{M}{1+r_L}\right)\left\{\frac{F^B}{S^A}(1+r^*_D)-(1+r_L)\right\}\cdot \left[\left(\frac{1}{(1+r_L)^{i-1}}\right)\cdot\left\{\frac{F^B}{S^A}(1+r^*_D)+\theta\left\{\frac{F^B}{S^A}(1+r^*_D)-(1+r_L)\right\}\right\}^{i-1}\right] \tag{5.23}$$

The summation over the first n consecutive iterations of arbitrage then yields the following amount of total profits:

$$\pi^{(+)0}_{Tt(16n)} \equiv \sum_{i=1}^{n}\pi^0_{Tt(i)} = \left(\frac{M}{1+r_L}\right)\left\{\frac{F^B}{S^A}(1+r^*_D)-(1+r_L)\right\}\times \sum_{i=1}^{n}\left(\frac{1}{(1+r_L)^{i-1}}\right)\left\{\frac{F^B}{S^A}(1+r^*_D)+\theta\left\{\frac{F^B}{S^A}(1+r^*_D)-(1+r)\right\}\right\}^{i-1} \tag{5.24}$$

Note here that the sum total of n rounds of consecutive arbitrage in the market gives rise to the magnification of the initial arbitrage profits by the factor of:

$$\sum_{i=1}^{n}\left(\frac{1}{(1+r_L)^{i-1}}\right)\left\{\frac{F^B}{S^A}(1+r^*_D)+\theta\left\{\frac{F^B}{S^A}(1+r^*_D)-(1+r)\right\}\right\}^{i-1}.$$

If $0 < [1/(1 + r_L)][(F^B/S^A)(1 + r^*_D) + \theta\{(F^B/S^A)(1 + r^*_D) - (1 + r_L)\}] < 1$, the profit function is convergent to its upper bound, which is defined by:

$$\pi^{(+)0T}_{t(164)} = \left(\frac{M}{1+r_L}\right)\{\frac{F^B}{S^A}(1+r^*{}_D)-(1+r_L)\}\cdot\left(\frac{1}{1-\hat{\alpha}_t\hat{\beta}_t}\right),$$

$$\text{where } \hat{\alpha}_t = \frac{1}{1+r_L} \text{ and } \hat{\beta}_t = \left[\frac{F^B}{S^A}(1+r^*_D)+\theta\,[\frac{F^B}{S^A}(1+r^*_D)-(1+r_L)]\right].$$

Table 5.1a
Arbitrage-induced total profits with no transaction costs and no borrowing against the equity positions: profit multiplier in each round (column 3), profits in each round (column 4), cumulative profit multiplier (column 5), and cumulative profits (column 6).

i	θ	$\mu_{(i)}$	$\pi^0_{(n)}$	$\mu^*_{(n)}$	$\pi^{(+)0}_{T\iota(1\to n)}$
1	0	1.00	$6,236.60	1.00	$6,236.60
2	0	1.01	6,275.50	2.01	12,512.10
3	0	1.01	6,314.64	3.02	18,826.74
4	0	1.02	6,354.02	4.04	25,180.76
5	0	1.03	6,393.65	5.06	31,574.41
6	0	1.03	6,433.52	6.09	38,007.93
7	0	1.04	6,473.65	7.13	44,481.58
8	0	1.04	6,514.02	8.18	50,995.59
9	0	1.05	6,554.64	9.23	57,550.24
10	0	1.06	6,595.52	10.29	64,145.76
20	0	1.13	7,018.60	21.23	132,406.20
25	0	1.16	7,240.21	26.96	168,161.26
100	0	1.85	11,541.42	138.24	862,135.12
∞	0	∞	∞	∞	∞

Following the earlier procedure, let $\mu_{Ti}(i)/(1/(1+r_L)^{i-1}.\ \{F^B/S^A)(1+r^*_D)+\theta\{F^B/S^A(1+r^*_D)-(1+r)\}\}^{i-1}$, and $\mu^+_{Ti}(n)/(1-(\hat{\alpha}\hat{\beta})^n/1-(\hat{\alpha}_i\hat{\beta}_i)$ the profit multipliers at ith round and cumulatively for first n rounds, respectively. The numerical values of these multipliers are given in Table 5.1.

ARBITRAGE PROFITS WITHOUT TRANSACTIONS COSTS ONCE AGAIN

Here we resurrect the earlier section that dealt with profit multiplier in the absence of transaction costs, once again with further modification and obvious generalization. We have already noted that once the present value of first round of profit (π_1^0) is recognized, the investor puts π_1^0 into arbitrage and then generates the following amount of profits in the second round:

Table 5.1b
Arbitrage-induced total profits with no transaction costs, and 25 percent borrowing against the equity positions: profit multiplier in each round (column 3), profits in each round (column 4), cumulative profit multiplier (column 5), and cumulative profits (column 6).

i	θ	$\mu_{(i)}$	$\pi^0_{(n)}$	$\mu^*_{(n)}$	$\pi^{(+)0}_{T\iota(1\to n)}$
1	0.25	1.00	$6,236.60	1.00	$6,236.60
2	0.25	1.01	6,285.22	2.01	12,521.83
3	0.25	1.02	6,334.22	3.02	18,856.05
4	0.25	1.02	6,383.60	4.05	25,239.65
5	0.25	1.03	6,433.37	5.08	31,673.02
6	0.25	1.04	6,483.52	6.12	38,156.54
7	0.25	1.05	6,534.06	7.17	44,690.60
8	0.25	1.06	6,585.00	8.22	51,275.60
9	0.25	1.06	6,636.34	9.29	57,911.94
10	0.25	1.07	6,688.07	10.36	64,600.01
20	0.25	1.16	7,228.13	21.55	134,416.48
25	0.25	1.20	7,514.30	27.48	171,411.21
100	0.25	2.16	13,453.28	150.59	939,171.82
∞	0.25	∞	∞	∞	∞

$$\pi_2 = \left(\left(\frac{\pi_1^0}{S}\right)(1+r^*)F - \pi_1^0(1+r)\right) = \pi_1^0\left\{\left(\frac{F}{S}\right)(1+r^*)-(1+r)\right\},$$

the present value of which is then:

$$\pi_2^0 = \left[\frac{M}{(1+r)^2}\right]\left\{\left(\frac{F}{S}\right)(1+r^*)-(1+r)\right\}^2 \qquad (5.25)$$

if pure arbitrage profit scenario is considered. Otherwise, the investor computes the total profit level (in the arbitrage-induced sense) to be:

$$\pi^+_{T(2)} = \left(\frac{\pi_1^0}{S}\right)(1+r^*)F,$$

Table 5.1c
Arbitrage-induced total profits with transaction costs, and no borrowing against the equity positions: profit multiplier in each round (column 3), profits in each round (column 4), cumulative profit multiplier (column 5), and cumulative profits (column 6).

i	θ	$\mu_{Tc(i)}$	$\pi^{\circ}_{Tc(i)}$	$\mu^{*}_{Tc(n)}$	$\pi^{(+)0}_{Tc(1\to n)}$
1	0	1.00	$931.20	1.00	$931.20
2	0	1.05	974.59	2.00	1,863.26
3	0	1.09	1,020.02	3.00	2,796.58
4	0	1.15	1.067.56	4.01	3,729.99
5	0	1.20	1,117.31	5.01	4,664.66
6	0	1.25	1,169.38	6.01	5,600.19
7	0	1.31	1,223.89	7.02	6,536.60
8	0	1.37	1,280.93	8.03	7,473.89
9	0	1.44	1,340.63	9.03	8,412.04
10	0	1.50	1,403.11	10.04	9,351.07
20	0	2.37	2,212.71	20.18	18,789.58
25	0	2.97	2,778.69	25.28	23,541.88
100	0	88.92	84,642.91	104.75	97,545.35
∞	0	∞	∞	∞	∞

which, as noted earlier, is:

$$\pi^{(+)0}_{T(2)} = \frac{\frac{\pi^0_1}{S}(1+r^*)F}{(1+r)}$$

$$= \left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)(1+r^*)-(1+r)\right\}\left\{\frac{1}{(1+r)}\right\}\left[\left(\frac{F}{S}\right)(1+r^*)\right], \tag{5.26}$$

and similarly,

$$\pi^{(+)0}_{T(3)} = \left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)(1+r^*)-(1+r)\right\}\left\{\frac{1}{(1+r)^2}\right\}\left[\left(\frac{F}{S}\right)(1+r^*)\right]^2 \tag{5.27}$$

Table 5.1d
Arbitrage-induced total profits with transaction costs, and 25 percent borrowing against the equity positions: profit multiplier in each round (column 3), profits in each round (column 4), cumulative profit multiplier (column 5), and cumulative profits (column 6).

i	θ	$\mu_{Tc(i)}$	$\pi^{\circ}_{Tc(i)}$	$\mu^{*}_{Tc(n)}$	$\pi^{(+)0}_{Tc(1\to n)}$
1	0.25	1.00	$931.20	1.00	$931.20
2	0.25	1.05	974.59	2.00	1,863.47
3	0.25	1.10	1,020.02	3.00	2,796.84
4	0.25	1.15	1,067.56	4.01	3,731.29
5	0.25	1.20	1,117.31	5.01	4,666.83
6	0.25	1.26	1,169.38	6.02	5,603.45
7	0,25	1.31	1,223.89	7.02	6,541.17
8	0.25	1.38	1,280.93	8.03	7,479.98
9	0.25	1.44	1,340.63	9.04	8,419.88
10	0.25	1.51	1,403.11	10.05	9,360.88
20	0.25	2.39	2,212.71	20.22	18,831.29
25	0.25	3.00	2,778.69	25.35	23,607.97
100	0.25	92.92	4,642.91	105.99	98,694.75
∞	0.25	∞	∞	∞	∞

and

$$\pi^{(+)0}_{T(i)} = \left[\frac{M}{(1+r)}\right]\left\{\left(\frac{F}{S}\right)(1+r^*)-(1+r)\right\}\left\{\frac{1}{(1+r)^{i-1}}\right\}\left[\left(\frac{F}{S}\right)(1+r^*)\right]^{i-1} \tag{5.28}$$

Note that in all these arbitrage activities, the investor thus far only puts in the present value of the generated arbitrage profits into the next round of covered arbitrage. In reality, she can (and rationally should) use the original amount M and $\pi_i^0(1 + \theta)$ for the $(i + 1)$ round of arbitrage ($i = 1, 2, 3, \ldots, n$). It is instructive, therefore, that we modify the earlier section that dealt with profit multiplier in the absence of transaction costs. The first round of arbitrage profits is obviously the same as before, but let it be rewritten (in this modified environment) as follows:

$$\hat{\pi}_1^0 = M\left[\frac{F}{S}\frac{1+r^*}{1+r} - 1\right] = M\alpha$$

where $\alpha \equiv \left[\frac{F}{S}\frac{1+r^*}{1+r} - 1\right]$,

and now, the second round is as follows:

$$\hat{\pi}_2 = \left(\frac{M + \hat{\pi}_1^0(1+\theta)}{S}\right)(1+r^*)F - (M + \theta\hat{\pi}_1^0)(1+r),$$

whence:

$$\hat{\pi}_2^0 = M + \theta\hat{\pi}_1^0\left[\frac{F}{S}\frac{1+r^*}{1+r} - 1\right] = M\alpha[1 + \{1+\alpha(1+\theta)\}].$$

Similar operations yield in the third round of iterative arbitrage:

$$\hat{\pi}_3^0 = M + \theta\hat{\pi}_2^0\left[\frac{F}{S}\frac{1+r^*}{1+r} - 1\right] = M\alpha\left[1 + \{1+\alpha(1+\theta)\}^2\right] + \{1+\alpha(1+\theta)\},$$ and in the ith round of iterative arbitrage:

$$\hat{\pi}_i^0 = M + \theta\hat{\pi}_{i-1}^0\left[\frac{F}{S}\frac{1+r^*}{1+r} - 1\right] = M\alpha\left[1 + \{1+\alpha(1+\theta)\}^{i-1}\right] + \{1+\alpha(1+\theta)\}^{i-2}.$$

Let $A \equiv \{1 + \alpha(1 + \theta)\}$. A close look at the expressions of π_1^0 for $i = 1, 2, 3, 4, \ldots, n$ reveals the following:

$\hat{\pi}_1^0 = M\alpha(A^0)$,

$\hat{\pi}_2^0 = M\alpha(A^0 + A^1)$

$\hat{\pi}_3^0 = M\alpha(A^0 + A^1 + A^2)$

$\hat{\pi}_4^0 = M\alpha(A^0 + A^1A^2 + A^3)$, and

.

.

$\hat{\pi}_n^0 = M\alpha(A^0 + A^1A^2 + A^3 + \ldots + A^{n-1})$.

The successive n rounds of iteration in arbitrage in the currency market then create the cumulative profits in the amount of:

$$\hat{\pi}(1 \to n) \equiv \sum_{i=1}^{n} \hat{\pi}_i^0 = M\alpha[n + (n-1)A + (n-2)A^2 + (n-3)A^3 + \ldots A^{n-1}]$$

for $n - i \geq 0$ and $i \geq 1$. Here $M\alpha$ is the initial profit ($\pi_i^0 \equiv \pi_i^0$), and $[n + (n-1)A + (n-2)A^2 + (n-3)A^3 + \ldots + A^{n-1}]$ is the cumulative multiplier $\mu(1 \to n)$ in this modified analytical framework.

Now all the previous sections can be appropriately modified with and without transaction costs.

Covered Interest Triangular Arbitrage (CITA):

(Pure Arbitrage) Profits under CITA

Next, we examine the scope of the covered interest arbitrage in triangularity. For the sake of simplicity, assume away transaction costs in currency and capital (or money) market. We again start off with a scenario like the following: an investor begins the process with U.S. dollars (let it be the first currency), which he converts into British pound (second currency), invests the converted pound amount in U.K. at the British rate of interest r_2. The pound amount generated in the investment process is then converted into Japanese yen (third currency), and the yen amount is then invested in the Japanese market. Finally, the newly created yen value is then converted back into U.S. dollars. Let us denote different terms as follows:

M = (original) amount of, say, U.S. dollars borrowed for investment;

r_1 = U.S. interest rate;

r_2 = British interest rate;

r_3 = Japanese interest rate;

S_{21} = spot rate of exchange of British pounds in terms of U.S. dollars (second currency in terms of first currency);

S_{32} = spot rate of exchange of Japanese yens in terms of British pounds (third currency in terms of second currency);

S_{31} = spot rate of exchange of Japanese yens in terms of U.S. dollars (third currency in terms of first currency);

F_{31} = forward rate of exchange of Japanese yen in terms of U.S. dollars.

Under the scenario, the investor's present value of covered interest triangular arbitrage profits ($P^+_{1(0:CITA)}$) can be computed as follows:

$$P^+_{1(0:CITA)} = \left(\frac{M}{S_{21}}\left[(1+r_2)\frac{1}{S_{32}}(1+r_3)F_{31} - M(1+r_1)\right]\frac{1}{1+r_i}\right) =$$
$$M\frac{1}{(1+r_1)}\left[\frac{F_{31}}{S_{32}S_{21}}(1+r_2)(1+r_3)-(1+r_1)\right], \tag{5.29}$$

and then,

$$P^+_{i(0:CITA)} = M\frac{1}{(1+r_1)^i}\left[\frac{F_{31}}{S_{32}S_{21}}(1+r_2)(1+r_3)-(1+r_1)\right]^i, \; i=1,\, 2,\ldots n. \tag{5.30}$$

The cumulative profit in the first n rounds is then defined by ($P^{+*}_{n:CITA}$):

$$P^{+*}_{n:CITA} = \sum_{i=1}^{n} P^+_{i(0:CITA)} = M\frac{1}{(1+r_1)^i}\left[\frac{F_{31}}{S_{32}S_{21}}(1+r_2)(1+r_3)-(1+r_1)\right]^i = M\lambda\left[\frac{1-\lambda^n}{1-\lambda}\right],$$

where $\lambda \equiv \frac{1}{1+r_1}\left[\frac{F_{31}}{S_{32}S_{21}}(1+r_2)(1+r_3)-(1+r_1)\right]$. Here $\left[\frac{1-\lambda^n}{1-\lambda}\right]$ is the pure arbitrage profit multiplier under CITA ($\mu^+_{\infty(0:CITA)}$). Since $0<\lambda<1$, for $n=\infty$, (5.31)

$$\mu^+_{\infty(0:CITA)} = \frac{1}{1-\lambda}.$$

Let us now use some hypothetical market data: $r_1 = 10\%$, $r_2 = 9.5\%$, $r_3 = 9.75\%$, $S_{21} = 2.00$, $S_{31} = 0.0098$, $S_{32} = 0.0049$, $F_{31} = 0.0105$, and $M = \$1{,}000{,}000$. With these data then, one gets the following computed values: $P^+_{1(0)}$ = \$170,547.90 (as compared to $P_{1(0)}$ = \$70,113.50), $\mu^+_{\infty(0:CITA)} = 1.2056151$ (compared to $\mu_\infty = 1.0754001$). It appears that magnification effect of arbitrage profit under covered interest triangular arbitrage (CITA) usually is more pronounced than it is under simple covered interest arbitrage (CIA). It is intuitively clear that if there is an additional gain from a movement from a two-currency to three-currency in arbitrage context, CITA must be superior to CIA. To ascertain the general validity of such ranking, one must compare CITA with n currencies *vis-a-vis* CITA with $(n-1)$ currencies where $n \geqslant 3$. If the former is superior to the later, one should use CITA instead of CIA in exploiting market opportunities.

Arbitrage-Induced Profits under CITA:

As in an earlier section, we can now eliminate the interest deductions from round two onward in the total profit calculation under covered interest triangular arbitrage. The first round, of course, remains the same as (5.31). It is easily then computed that the present value of the profit level on ith iteration ($i > 1$) under CITA ($\tilde{P}_{i(0:CITA)}$) is as follows:

$$\tilde{P}_{i(0:CITA)} = \frac{1}{(1+r_1)^{i-1}}\left[\frac{F_{31}}{S_{32}S_{21}}(1+r_2)(1+r_3)\right]^{i-1}\cdot\theta, \tag{5.32}$$

$$\text{where } \theta = M\frac{1}{(1+r_1)}\left[\frac{F_{31}}{S_{32}S_{21}}[(1+r_2)(1+r_3)-(1+r_1)]\right].$$

The cumulative profits over n iterations ($\tilde{P}^+_{n:CITA}$) are measured then by the following:

$$\tilde{P}^*_{n(CITA)} = M\frac{1}{1+r_1}\left[\frac{F_{31}}{S_{32}S_{21}}(1+r_2)(1+r_3)-(1+r_1)\right]\sum_{i=1}^{n}\frac{1}{(1+r_1)^{i-1}}\left[\left[\frac{F_{31}}{S_{32}S_{21}}(1+r_2)(1+r_3)\right]^{i-1}\right]. \tag{5.33}$$

Here, (pure) profit multiplier ($\mu^+_{n(0:CITA)}$), profit level ($P^+_{n(0:CITA)}$) under CITA, (arbitrage-induced) profit multiplier ($\tilde{\mu}^+_{n(0:CITA)}$), and profit levels ($\tilde{P}^+_{n(0:CITA)}$) under different values of n under CITA are given by Table 5.2:

Next, one may wonder why an investor should begin the arbitrage process with \$1 million in place of \$1 billion or \$10 trillion if the arbitrage opportunities exist. In this chapter, that is not the issue for discussion. We must, however, point out that we never attempt to measure the optimum

Table 5.2
Pure arbitrage profits, arbitrage-induced profits, and different multipliers under CITA with M = \$1,000,000; r_1 = 10%; r^2 = 9.5%; r_3 = 9.75%; S_{21} = 2.000; S_{31} = 0.0098; S_{32} = 0.0049; F_{31} = 0.0105.

n	$\mu^+_{n(0:CITA)}$	$P^+_{n(0:CITA)}$	$\tilde{\mu}^+_{n(0:CITA)}$	$\tilde{P}^+_{n(0:CITA)}$
1	1	\$170,547.70	1	\$170,547.70
2	1.1705477	\$199,634.20	2.2151139	\$377,782.58
3	1.1996341	\$204,594.84	3.6917016	\$629611.21
4	1.2045948	\$205,440.87	5.4859309	\$935,612.89
5	1.2054409	\$205,585.17	7.6661683	\$1,307,447.30
10			27.976086	\$4,771,257.10
20			224.33316	\$38,259,504
25			601.99015	\$120,668,035
∞	1.2056148	\$205,614.84	4	4

level of investment. One may look at it factually that the amount the investor begins with is the maximum amount he has (or can borrow from a bank) at the point when he engages in his arbitrage operation, and thus one can take that amount as M in this theoretical paradigm. Another point must be raised here next. It is often stated that since interest rate parity exists as a condition for market equilibrium, and hence the arbitrage profit does not exist in the very first round, then arbitrage profit as well as its multiplier must be zero. Having seriously considered this point, we have noted almost always that at any point in time, there exists a set of interest rates, different from a pair that is consistent with interest rate parity.[5] In other words, one can almost surely find a constellation of interest rates for which covered interest arbitrage veritably exists.

At a more general level, we must now suggest that one may follow the earlier procedure to bring all types of transaction costs into the analytical framework. With several experiments with real-market data collected in several periods, we note that covered interest triangular arbitrage (CITA) appears to be a better investment strategy than simple design of covered interest arbitrage (CIA), although further econometric testing should be performed to put real emphasis to our tentative conclusion in the real world. Further extensions are left for the readers and interested practitioners who may decide to play in foreign exchange markets. One should note that although, in this paper, we have worked out CITA activities with three currencies, it is neither difficult nor less useful to deal with any arbitrary number of currencies (greater than three) with this analytical procedure outlined here. In fact, the mathematical generalization of equation 5.33 is as follows:

$$\tilde{P}^{+}_{n:\mathrm{CITA}} = \tilde{\theta}\sum_{i=1}^{n}\frac{1}{(1+r_j)^{i-1}}\left[\frac{F_{mj}}{\prod_{j=1,j\neq m}^{m} S_{j+1,j}}\prod_{j=1}^{m}(1+r_{j+1})\right],$$

$$\text{where } \tilde{\theta} = M\frac{1}{(1+r_j)}\left[\frac{F_{mj}}{\prod_{j=1,j\neq m}^{m} S_{j+1,j}}\prod_{j=1,j\neq m}^{m}(1+r_{j+1})-(1+r_j)\right], \tag{5.34}$$

$S_{j+1,j}$ is the rate of exchange of the $(j+1)$th currency in terms of the jth currency. Empirical testing of this multidimensional profit formula is one as complicated as it may appear at first sight.

SOME OBSERVATIONS

Although it is often contended that the dynamics of market competition wipe out arbitrage opportunities very quickly, and hence iterative arbitrage does not make much sense in view of the physical speed of each transaction, in the context to today's technology, it should be noted that one can make about one million calculations and 10,000 evaluations in 1 second through a computer. One should also note that market quotations last, on average, between 10 seconds and 4 minutes, depending on the time of the day, trading volume, and the currencies concerned. Having factored in all this information, it is quite feasible that multiple rounds of arbitrage operations can be performed with any market data remaining frozen for a few seconds or minutes. At this stage we must bring out the fact that to make such a series of transactions the arbitrageur must have the arrangement with her bank for digitized signature even though it requires a side collateral commitment of the arbitrageur's line of credit, which she knows she will not tie up in any real sense. A prior discussion with the bank is a good step in the logistical design for such market play.

One may contend that since profits out of the first round of arbitrage are obtained only at the end of, say, one year from that day, how is this investor getting funds for the second, the third, and other rounds of market plays? Note here that π_1 is a sure amount of money made by the investor without taking any risk, and any bank should recognize this amount the investor makes at the end of one year from now. If that is a common knowledge of the investor as well as that of the bank, it is equally recognizable that this investor has π^0_1 / $\pi_1/1 + r$ now—and it is now her equity position, which she can legitimately utilize with a prior discussion with her banker. Finally, it is worth noting, particularly against the backdrop of the common belief that markets are so well aligned that scope for arbitrage is nonexistent in reality, that in the currency market almost always one can find arbitrage opportunity. Note that although spot and forward rates are *usually* defined at a point in time, and corresponding to those defined rates a set of domestic and foreign interest rates will yield $\pi_1 = 0$, one can always find another set of interest rates, which generates π_1 ? 0. This clearly signifies that arbitrage opportunity is a viable and feasible strategy in the foreign exchange market more often than not. Additionally, it should be pointed out that one who watches real-time data can easily recognize that quotations on spot and forward rates by different banks and/or dealers are not always same at the same instant. So, on that front one may find the scope for arbitrage. Finally, we should note that if one round of arbitrage act is undertaken, it may appear that arbitrage profit is negligible, and in that sense one may conclude that arbitrage opportunity is virtually nonexistent. But as the Table 5.1 shows, iterations, and cumulation make arbitrage trading very significant in real life.

EMPIRICAL EVIDENCE

Using *Reuters* real-time data screen and checking many of their quotes with number of banks, the values of π's and μ's for different iteration i and different values of leverage θ have been computed. We compiled data for six weeks from June 19, 1995 through July 31, 1995. Out of those intra-day data, randomly chosen usually three times a day from *Reuters* data screen, we have made 100 sets of computations. As expected, computed values are different for different sets, and here we choose the set that lies within the median set of 5, and the geometric mean of these median sets has been used to calculate the entries in Tables 5.1c and 5.1d under different assumed values of θ.

Tables 5.1c and 5.1d presents the computed profits and profit multipliers in the presence of transaction costs in both currency and money markets. *Ask* and *bid* quotes in the foreign exchange markets and *deposit* and *lending* rates of interest were taken from the *Reuters* screen three times a day (around 10:00 A.M., 11:30 A.M., and 2:30 P.M.) within the same time period (June 19, 1995 through July 31, 1995), and almost a third of these intra-day quotes were verified with a number of banks. Here we have $S^A = 0.2028$, $F^A = 0.2037$, $r_B = 0.0454$, $r_D = 0.0444$, $r^*_B = 0.0440$, $r^*_D = 0.0438$, $M = \$1{,}000{,}000$, $T = 360$, $\theta = 0, 0.25, 0.5, 0.75$, and $i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 25, 100, \infty$. In the presence of transaction costs, profit measures are substantially reduced at each level of iteration, and hence cumulative measures are reduced as a result also.

CONCLUSION

We note through our computations with real-time data that the standard academic prognosis that arbitrage is hardly profitable because the markets are very well-aligned stands to be partly correct and partly incorrect. If \$1,000,000 generates \$931.20, which is not a significant profit (see Table 5.1a), one can hardly consider arbitrage as a meaningful investment instrument. Under transaction costs, both strategy (i) and strategy (ii) give negative profits 20 percent of the time in our data sample with deutsche mark *vis-a-vis* U.S. dollar. Of course, in those situations, no trader would engage in arbitrage, and hence, profit possibilities would simply be zero (not negative). It is in this sense one may agree with the existing academic research that tends to suggest that arbitrage is more of an illusion than a veritable reality. Yet, arbitrage-induced cumulative total profits, which are significant in magnitudes, have not been pointed out earlier. We find that arbitrage still provides a very meaningful window of opportunity for profit making without risk in currency market, where iterative arbitrage activities are undertaken under a skillful programming hand with maximum speed of transactions. Cumulative profits in Table 5.1 are quite eloquently demonstrative of that reality.

NOTES

The authors express their sincere appreciation to the referee, and Peter Zhang, C.F. Lee, Geoffrey Booth, Michael Sullivan, and Shmuel Hauser for useful and constructive comments on the previous draft. They are also indebted to K.C. Han, Philippe Givry, and David Suk for their computational assistance.

1. See, for instance, Ghosh (1997) on this issue.
2. See other citations in the *References* on this score.
3. Here r and r^* are the annual interest rates. If one has to get to the interest rate matching, say, a forward contract maturing in 90 days, then the investor should divide annual interest rates by 4 to arrive at the three-month interest rate of the United States and of the other country. One should note that the investor can choose any compounding or discounting interval, even continuous compounding or discounting for computations, and profit measures as a result will get modified.
4. It should be clearly noted at this point that if equation 5.14 assumes negative value, it does not automatically signify positive profit condition under investment strategy (ii). It is instructive therefore that we define profit measures under different arbitrage iterations when the investor is borrowing, say, N French francs at r^*_L, converting the amount into U.S. dollars at the spot rate of exchange (S^B), investing the converted dollar amount in the U.S. market at r_D, selling the dollars at the forward rate, and finally paying off the original debt and the accrued interest on that debt. In this case, her profits in the first round of arbitrage ($\pi^*_{t(1)}$) are computed as follows:

$$\begin{aligned}\pi^*_{t(1)} &= NS^B(1 + r_D)/F^A - N(1 + r^*_L)\\ &= N\{(S^B/F^A)(1 + r_D) - (1 + r^*_L)\},\end{aligned} \tag{5.34}$$

and therefore, the present value of the amount is:

$$\pi^{*(0)}_{t(1)} = [N/(1 + r_L)]\{(S^B/F^A)(1 + r_D) - (1 + r^*_L)\}. \tag{5.35}$$

On the ith iteration, then profits are defined by:

$$\pi^{*(0)}_{t(i)} = [N/(1 + r^*_L)^i]\{(S^B/F^A)(1 + r_D) - (1 + r^*_L)\}^i. \tag{5.36}$$

Note here that (5.14) and (5.34) are not *necessarily* of opposite signs[3].

5. In real life, some of these results have been used, and it is inappropriate to mention anything beyond this simple statement.

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