CHAPTER 6

THE INTERNATIONAL PARITIES

In this chapter we explain the concept and the conceptual necessities and relevance of all of the celebrated parity statements in international finance from the point of view of the participants in international markets that essentially involve foreign exchanges. There are four basic parities here to bring out and elaborate on. These parities are:

A. THE INTEREST RATE PARITY (OR THE COVERED INTEREST PARITY)

B. THE PURCHASING POWER PARITY (THE LAW OF ONE PRICE)

C. THE FISHER OPEN PARITY

D. THE FORWARD FUTURE-SPOT PARITY

It should be noted, however, that we have brought out another parity - namely, **The Call-Put Forward Parity** in chapter 4.

A. THE INTEREST RATE PARITY

A.1 Where to Invest In?

In this chapter, we begin by asking the question: if you have the choice between investing in the home economy and in the foreign economy with the same level of ease (or difficulty) and with no (or same) risk, where will you invest in? To answer this simple question, assume you have an investible fund of $100,000. and you have the following data for your decision-making:

Current Spot rate of exchange (R_s^0) = 2

Currently, 1-year forward rate of exchange (R_f^0) = 2.15

home (domestic) rate of interest (r_H) = 10%

foreign rate of interest (r_F) = 9.5%

Obviously, you have two possible choices: (i) invest at home at (10% $\equiv r_H$), or (ii) convert your investible dollars into foreign currency (say, pound sterling) at the current spot rate of exchange (2 $\equiv R_s^0$) and then invest the pound sterling amount at the foreign rate of return (9.5% $\equiv r_F$). In this second choice, you must do one

more thing to stay risk-free in the sense that any possible fluctuation in foreign exchange rate a year from today will not be your concern at all. To do so, you must sell today that amount of pound sterling at the forward rate which you calculate your pound sterling amount will be a year from today.

Let us do the exercise in numerical terms. From choice (i) - that is, if you invest $100,000 today at 10%, you will have $100,000(1 + 0.10) = $110,000 at the end of one year from now, and the rate of return is 10%:

$$\left(\frac{\textit{amount at the end of the year - amount at the beginning of the year}}{\textit{amount at the beginning of the year}}\right)$$

$$\frac{\$110,000 - \$100,000}{\$100,000} = 10\%$$

If, on the other hand, you decide to go through with choice (ii), then you convert your \$100,000 at the current spot rate of exchange and have \$100,000/2 = £50,000, which is then invested at 9.5% to yield £50,000(1 + 0.095) = £54,750 at the end of one year. Since you know now that this will be the amount in pound sterling at the end of one year from today, you can enter into a forward contract to sell £54,750 at the current 1-year forward rate of 2.15 and lock in the dollar amount of 54,750 x 2.15 = \$117,712.50. This is the assured amount, - there is no risk or uncertainty about it.

It is clear now that choice (ii) is superior to choice (i) for you since the amount generated by choice (ii) is higher than that obtained by choice (i). Note, now, that the rate of return from choice (ii) is:

$$\frac{\$117,712.50 - \$100,000}{\$100,000} = 17.712\%$$

If the rate of return from both choices is identical, then you will be choice-neutral, and in that case you are at what is called the ***interest rate parity*** (or ***covered interest parity***). Let us elaborate on this point beyond these numerical calculations and comparisons.

Let the initial investible funds of the investor be \$X. If the investor decides to invest his funds in his domestic economy, he gets $X(1 + r_H) at the end of the year, and the rate of return is r_H. If his decision is to try his alternative choice, then he exchanges his $X and gets £X/$R_s^0$, which is then invested at r_F and thus turns the original amount into [£X/R_s^0](1 + r_F). The investor sells this [£X/R_s^0](1 + r_F) then at the forward rate and turns this foreign currency amount into $[£X/$R_s^0$](1 + r_F)R_f^0. His rate of return is:

$$\frac{\$\left[\frac{X}{R_S^0}(1 + R_F)\ R_F^0 - \$X\right]}{\$X}$$

$$= \frac{R_f^0}{R_s^0}(1 + r_F) - 1$$

$$= \frac{R_f^0 - R_s^0}{R_s^0} + r_F\left(\frac{R_f^0}{R_s^0}\right)$$

= foreign currency's forward premium (or discount) + (foreign interest rate) *times* (1 + foreign currency's forward premium (or discount))[1].

---

1. Since (R_f^0 - R_s^0)/R_s^0 = foreign currency's forward premium (or discount), R_f^0/R_s^0 = 1 + foreign currency's forward premium (or discount).

Now, one can see that if:

$$r_H > \frac{R_f^0}{R_s^0}(1 + r_F) - 1 \quad \text{or} \quad \frac{1 + r_H}{1 + r_F} > \frac{R_f^0}{R_s^0} \tag{A.1}$$

then the investor must invest in the home country. Similarly, if:

$$r_H < \frac{R_f^0}{R_s^0}(1 + r_F) - 1, \quad \text{or}$$

$$\frac{1 + r_H}{1 + r_F} < \frac{R_f^0}{R_s^0} \tag{A.2}$$

the investor must invest in the foreign country. In the borderline case,

$$r_H = \frac{R_f^0}{R_s^0}(1 + r_F) - 1 \quad \text{or}$$

$$\frac{1 + r_H}{1 + r_F} = \frac{R_f^0}{R_s^0} \tag{A.3}$$

the investor is indifferent in regard to his choices. The situation, portrayed by (A.3) - that is, the case of equality between the left-hand side and the right-hand side, is the case of *interest rate parity*. From (A.3), one can easily deduce (by subtracting 1 from both sides):

$$r_H - r_F = \frac{R_f^0 - R_s^0}{R_s^0} (1 + r_f) \tag{A.3*}$$

So, the deduced rule is as follows: **if the interest rate differential between the home country and the foreign country equals the foreign currency's forward premium (discount) times foreign interest rate plus 1, the investor is choice-neutral in terms of his decision to invest at home or abroad.** From (A.1) one then finds that if:

$$r_H - r_F > \frac{R_f^0 - R_s^0}{R_s^0} (1 + r_F) \tag{A.1*}$$

the investor should stay at home with his investment dollars, and if:

$$r_H - r_F < \frac{R_f^0 - R_s^0}{R_s^0} (1 + r_F) \tag{A.2*}$$

he should invest abroad.

Let us draw the following diagram (Figure 6.1) to depict the choices of our investor:

Figure 6.1

Here we measure interest rate differential (r_H - r_F) along the horizontal axis, and forward premium (discount) *times* one plus foreign interest rate [$\{R_f^0 - R_s^0)/R_s^0\}(1 + r_F)$] along the vertical axis. If the investor is at point, say Z (which means $r_H - r_F = 10\%$ and ($\{(R_f^0 - R_s^0)/R_s^0\}(1 + r_F) = 10\%$), then it is a situation of interest rate parity for the investor, - he should be indifferent. If he finds himself at point,say, T, he is at the following situation: $r_H - r_F = 10\%$, but $\{(R_f^0 - R_s^0)/R_s^0\}(1 + r_F) = 12\%$, which means he is not at parity, - and in this specific instance: $r_H - r_F < \{(R_f^0 - R_s^0)/R_s^0\}(1 + r_F)$ - which is the condition described by (A.2*). So, in this case, he must invest overseas. From this one can conclude that if the investor is above the 45°-line of interest rate parity, his choice should be to invest abroad. By similar reasoning, the investor should invest in the domestic economy if he is at any point below the 45°-line of interest rate parity.

At this point it is imperative that we bring out a fact of approximation which many people accept as a matter of convenience or simplicity. It is the concept called 'second order of small': the product of two or more fractions is smaller and often considered negligible (that is, effectively equal to zero). Because of this approximation

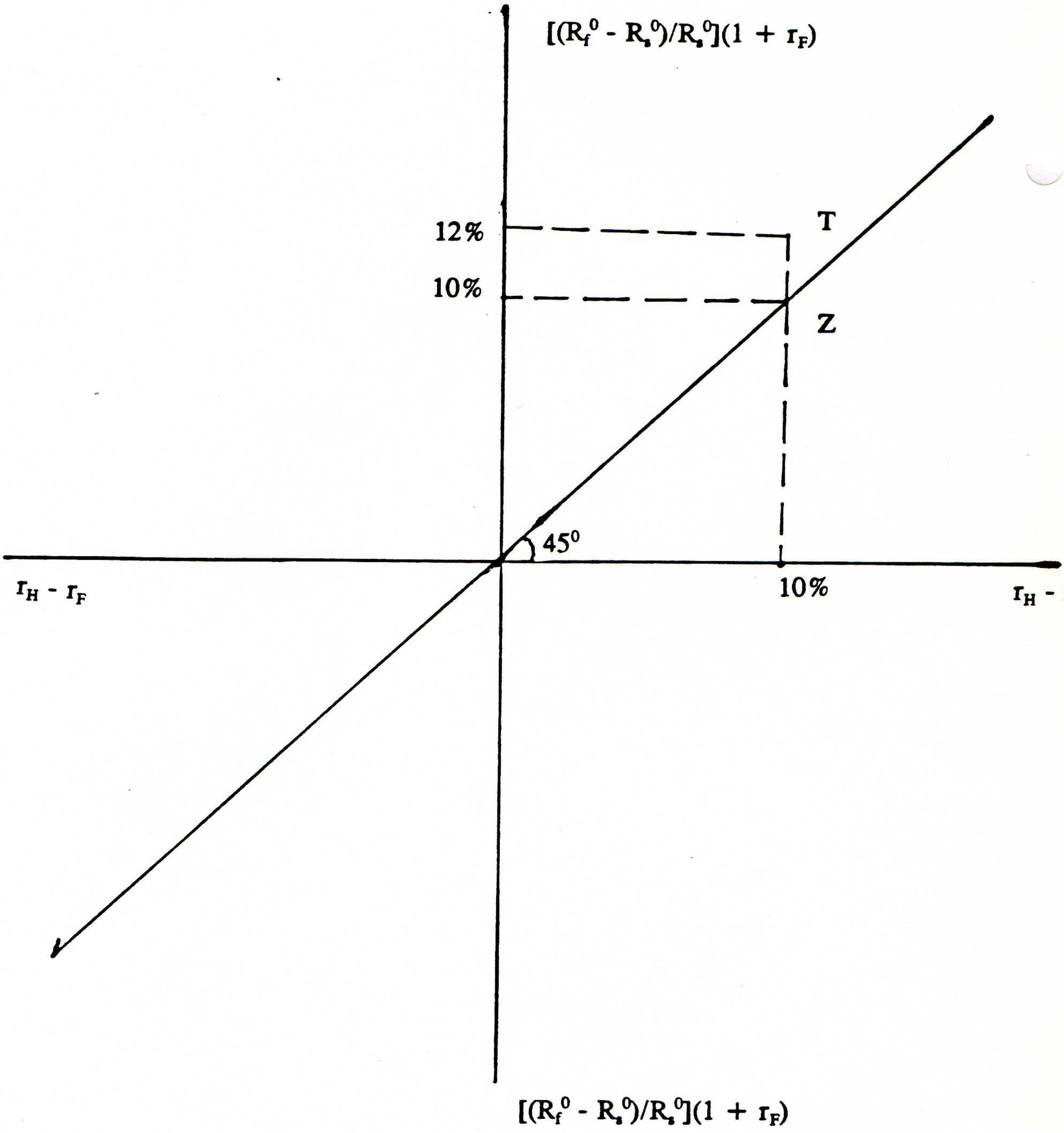


Figure 6.1

$$\frac{R_f^0 - R_s^0}{R_s^0}(1 + r_f) = \frac{R_f^0 - R_s^0}{R_s^0} + \frac{R_f^0 - R_s^0}{R_s^0} r_f$$

$$= \frac{R_f^0 - R_s^0}{R_s^0}$$

since $\frac{R_f^0 - R_s^0}{R_s^0} r_F \approx 0$ by the notion of the second order of small. With this approximation,

in many instances and in many textbooks, you may notice that the vertical axis of Figure 6.1 measures $\frac{R_f^0 - R_s^0}{R_s^0}$ in place of $\frac{R_f^0 - R_s^0}{R_s^0}(1 + r_F)$.

Try to focus on (A.3*) or (A.3) once again, and note that when both sides are equal, the investor may choose to invest in either the domestic market or in the foreign market without any gain or loss. If this condition of investor indifference is well-understood, then obviously one can draw the following diagram (Figure 6.2) to describe the investor's choices. Here, the vertical axis measures the ratio of current forward and spot rates of exchange (R_f^0/R_s^0). The straight line ZCMDY with the constant height of OM (and thus running parallel to the horizontal axis) measures $(1 + r_H)/(1 + r_F)$.

Figure 6.2

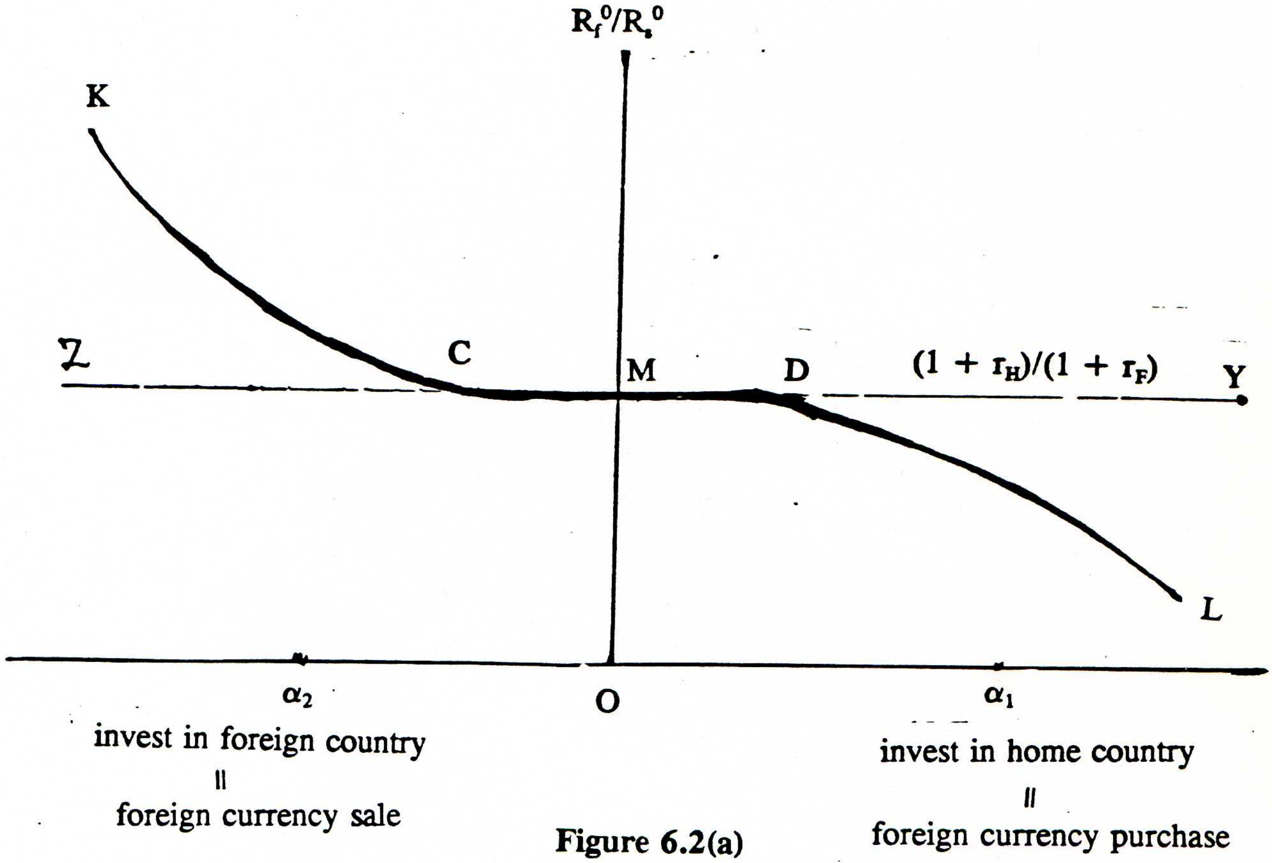


Figure 6.2(a)

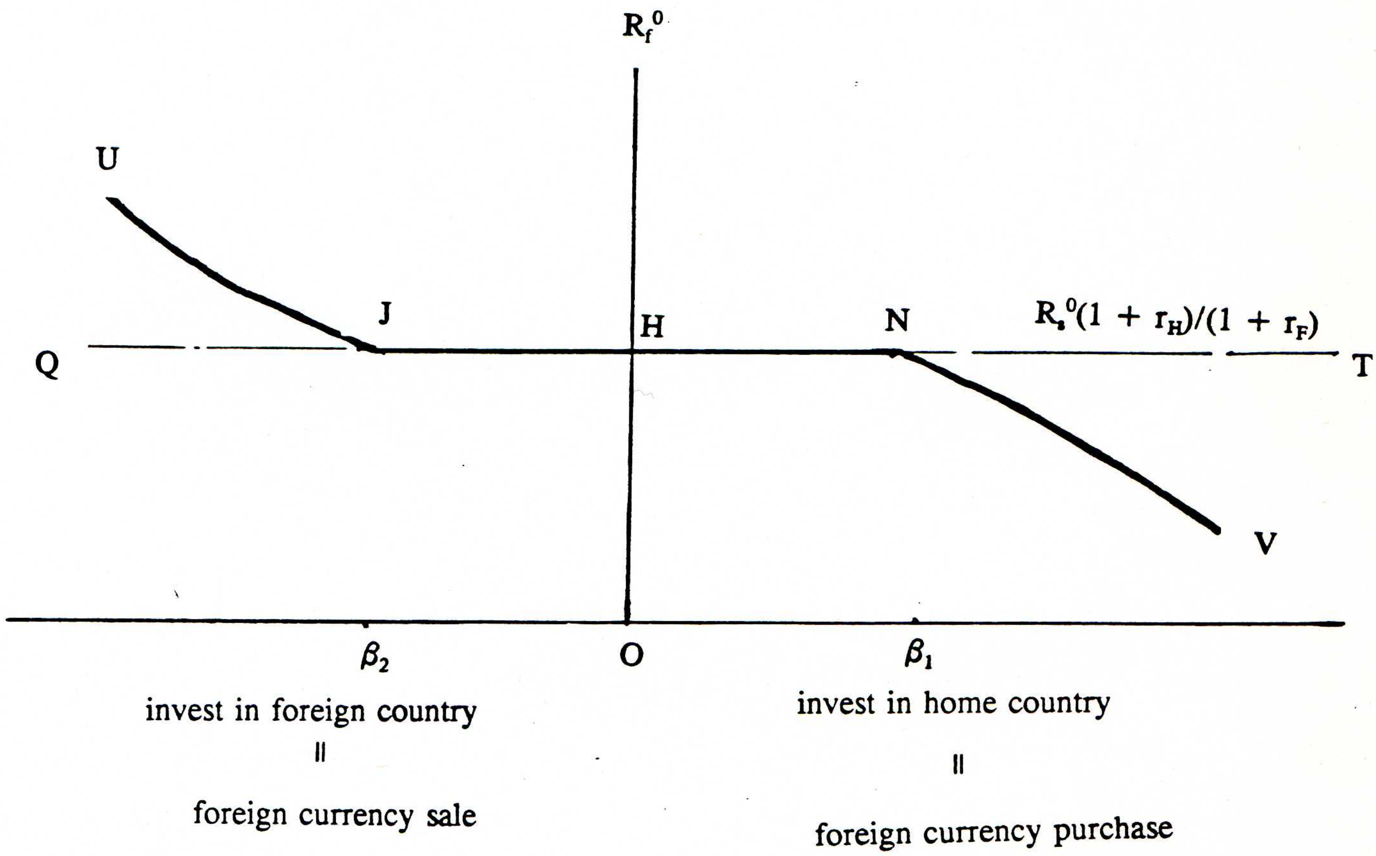


Figure 6.2(b)

Look at Figure 6.2(a). As long as $\frac{1 + r_H}{1 + r_F} = \frac{R_f^0}{R_s^0}$, the investor may do whatever he chooses to do, - that is, he may invest in the domestic market or he may invest in the foreign market. The curve KCMDL depicts just that. When the height of this curve equals $(1 + r_H)/(1 + r_F)$, the investor may invest in either market, and this is being described by the horizontal range CMD of the curve KCMDL. When the investor finds himself on the range CM in the range CMD (whose height is $(1 + r_H)/(1 + r_F)$), the investor invests in the foreign market, and when he is on the range of MD in the range CMD, he invests in home economy. But when the height of the curve KCMDL is lower than $(1 + r_H)/(1 + r_F)$, as at $0\alpha_1$, the investor must invest in the home country. Similarly, at $0\alpha_2$ the height of the curve KCMDL is higher than $(1 + r_H)/(1 + r_F)$, and the investor must invest in the foreign economy. The reconstruction of the same results in a slightly different diagram yields Figure 6.2(b), in which the vertical axis now measures R_f^0. The picture and its portrayed messages are clear. Here the height of the straight line QJHNT running parallel to the horizontal axis measures $R_s^0(1 + r_H)/(1 + r_F)$. If $R_f^0 = R_s^0(1 + r_H)/(1 + r_F)$. then it is a matter of indifference for the investor on his choice of locale for investment, - he may choose either home country or foreign country without any additional gain or loss. However, if $R_f^0 > R_s^0(1 + r_H)/(1 + r_F)$. he should invest overseas, and in the opposite case ($R_f^0 < R_s^0(1 + r_H)/(1 + r_F)$. he should invest in his home country.

A bit more should be pointed out at this moment. So far, we have made a distinction by way of saying "home economy, country or market" and "foreign economy, country or market". The reason was simple. If you start off with dollars, the dollar economy is the home economy, and the other economy (in this illustrative exposition) is the foreign economy. It is, therefore, high time to point out that what is true for an American investor should be true for a British (≡ foreign) investor as well under a given set of financial data. That means, if the U.S. interest rate is 10% and the British interest rate is 9.5% and the spot and forward rates of the British pound in terms of U.S. dollars are 2 and 2.15, respectively, the superior decision for any investor (regardless of whether he is American or British) is to do the following: **If**

$$R_{US} - r_{UK} > \left(\frac{R_f^0 - R_s^0}{R_s^0} \right) (1 + r_{UK}) \qquad \text{invest in U.S.;}$$

$$r_{US} - r_{UK} < \left(\frac{R_f^0 - R_s^0}{R_s^0} \right) (1 + r_{UK}) \qquad \text{invest in U.K.;}$$

$$r_{US} - r_{UK} = \left(\frac{R_f^0 - R_s 0}{R_s^0} \right) (1 + r_{UK}) \qquad \text{invest in either U.S. or U.K.}$$

A.2 Where to Borrow From?

Now, the question as to where to borrow from must be duly addressed to. Recall that we started off with the assumption that you have \$100,000 to invest. If the question is slightly modified, and you are given the information that you can get \$100,000 (or £50,000) now at a cost of borrowing of 10% from a U.S. bank or 9.5% from a British bank, where will you borrow from? From the exercise we have performed, it is now clear that if you borrow at 10% and then go through choice (ii) of your investment strategy, you will make \$117,712.50, but the principal amount borrowed plus the accrued interest on it will be \$110,000 for paying off the debt. The net result is a gain of \$7,712.50, - which means a net rate of positive return on your investment strategy. Any other borrowing and investment decision will be inferior. A detailed analysis along these lines yields the following results:

i. **when $\{(1 + r_H)/(1 + r_F)\} < (R_f^0/R_s^0)$,**

investor should borrow home currency at domestic rate of interest (r_H), exchange the borrowed amount of home currency into foreign currency at spot rate of exchange (R_s^0), invest (lend) the foreign currency amount thus obtained in the foreign market at foreign rate of interest (r_F), and sell the foreign currency amount investor will receive at the end of the year at forward rate (R_f^0).

ii. **when $\{(1 + r_H)/(1 + r_F)\} > (R_f^0/R_s^0)$,**

investor should borrow foreign currency at foreign rate of interest (r_F), exchange the borrowed amount of foreign currency into home currency at spot rate of exchange

(R_s^0), invest (lend) the home currency amount thus obtained in the home market at home rate of interest (r_H), and buy the foreign currency amount investor will pay at the end of the year at forward rate (R_f^0).

If inequality (< or >) is replaced by equality (=) in case (i) or in case (ii) above, investor is back to choice-neutrality again. One can express these findings in terms of Figure 6.2, if right-hand and left-hand horizonal axis are now relabeled to read "foreign currency purchase" and "foreign currency sale", respectively. Basically, in all the cases in which

$$\{(1 + r_H)/(1 + r_F)\} \neq R_f^0/R_s^0,$$

investor is arbitraging up until the equality is restored. This is called ***covered interest arbitrage***.

A.3 Importer's (Exporter's) Strategies

Thus far, the spot rate and forward rate of exchange have been dealt with from an investor's point of view. Let us now reexamine the importance of spot and forward markets of exchange rates from the point of view of a trader who engages in either importing merchandise from a foreign country or exporting goods and services to a foreign country. Consider an importer who buys British goods worth £50,000 due in 1 year. He notes now that $R_s^0 = 2$, $R_f^0 = 2.15$, $r_H = 10\%$, and $r_F = 9.5\%$. In this case he can either buy today an amount of pound sterling that will be exactly £50,000 a year from today, or he can buy £50,000 at a 1-year forward rate. If he chooses the first alternative, then he borrows U.S. dollars at r_H (=10%) and buys the British pound with that dollar at spot market, then invests the amount in the British economy at r_F (= 9.5%), and finally pays his bill of £50,000 at the end of one year. In this situation, the domestic bank does the import financing. If, however, this importer chooses the forward cover for his import bill, he pays R_f^0 per British pound at the end of the year. Which is the cheaper cover: spot or forward? It is obvious now that if:

$$R_s^0 \left(\frac{1 + r_H}{1 + r_F}\right) = R_f^0,$$

importer should be choice-indifferent. When

$$R_s^0 \left(\frac{1 + r_H}{1 + r_F}\right) > R_f^0,$$

he should go through forward contract and if the reverse inequality holds, his choice should be to cover his import bill in the spot market. One can then restate these findings as follows:

If:

$$R_s^0 \left(\frac{1 + r_H}{1 + r_F}\right) = R_f^0, \qquad \text{foward cover}$$

$$R_s^0 \left(\frac{1 + r_H}{1 + r_F}\right) = R_f^0, \qquad \text{indifference}$$

$$R_s^0 \left(\frac{1 + r_H}{1 + r_F} \right) < R_f^0, \qquad \text{spot cover}$$

or alternatively, if:

$$\left(\frac{1 + r_H}{1 + r_F} \right) > \frac{R_f^0}{R_s^0}, \qquad \text{forward cover}$$

$$\left(\frac{1 + r_H}{1 + r_F} \right) = \frac{R_f^0}{R_s^0}, \qquad \text{indifference}$$

$$\left(\frac{1 + r_H}{1 + r_F} \right) < \frac{R_f^0}{R_s^0}, \qquad \text{spot cover}$$

appear to be the superior (cost-minimizing) choices.

Note that the *covered interest parity* or the absence thereof determines not only the investment strategy, - it determines the trade financing strategy as well.

B. THE PURCHASING POWER PARITY

In international markets a unit of a good sells for the same price when the price is measured in the same currency in perfectly competitive market condition which assumes full rationality of all market participants and does not admit of any market distortions or trade impediments such as tariffs or taxes, transport costs, and the like. What it means is that if 1 British pound today equals 2 U.S. dollars ($R_s^0 = 2$), then if a Jaguar currently costs £20,000 in U.K., it will cost $40,000 in U.S. That is:

$$P_H^0 = R_s^0 \, . \, P_F^0 \qquad (B.1)^0$$

where P_H^0 is the current price of a Jaguar in domestic monetary terms (U.S. dollars), P_F^0 is the current price for the same automobile in foreign currency (British pound), and R_s^0 is the spot rate of exchange (U.S. dollars per British pound). This is called the ***Law of One Price***. It holds, if it does, is due to international commodity arbitrage by traders. This is the purchasing power parity in its absolute version (PPP_A). From $(B.1)^0$ one then gets:

$$R_s^0 = \frac{P_H^0}{P_F^0} \qquad (B.2)^0$$

What is true *now* (that is, at the current period) should be true at period 1 from now (say, 1 year from today), and that means:

$$R_s^1 = \frac{P_H^1}{P_F^1} \qquad \text{(B.2)}^1$$

Combine $(B.2)^0$ and $(B.2)^1$, and obtain:

$$\frac{R_S^1}{R_S^0} = \frac{\frac{P_H^1}{P_F^1}}{\frac{P_H^0}{P_F^0}} = \frac{\frac{P_H^1}{P_H^0}}{\frac{P_F^1}{P_F^0}} \qquad \text{(B.3)}$$

Note that, by definition, $\{(P_H^1 - P_H^0)/P_H^0\} = i_H$ and $\{(P_F^1 - P_F^0)/P_F^0\} = i_F$, where i_H and i_F are the inflation rates at time 1 in the home and in the foreign economy, respectively. Since $\{(P_H^1 - P_H^0)/P_H^0\} = i_H$, one finds immediately:

$$\frac{P_H^1}{P_H^0} = 1 + i_H, \qquad \text{(B.4.1)}$$

and similarly,

$$\frac{P_F^1}{P_F^0} = 1 + i_F, \tag{B.4.2}$$

The substitutions of (B.4.1) and (B.4.2) in (B.3) yield:

$$\frac{R_S^1}{R_S^0} = \frac{1 + i_H}{1 + i_F} \tag{B.5}$$

Now, if you subtract 1 from both sides of (B.5), the relationship remains unchanged, and you get the following new expression:

$$\frac{R_S^1 - R_S^0}{R_S^0} = \frac{i_H - i_F}{1 + i_F^2} \tag{B.6}$$

Since the lefthand side of (B.6) measures the rate of change in spot rate of exchange (currency appreciation or depreciation), (B.6) defines the following:

rate of exchange rate appreciation (or depreciation) = inflation rate differential *divided by* 1 *plus* foreign inflation rate.

This is *purchasing power parity* in its relative version (PPP_R). Consider an example at this point. Mexico's inflation rate is 20 percent; U.S. inflation rate is 6 percent. From the relation (B.6), one should conclude that the Mexican peso will depreciate by 11.6 percent against the U.S. dollar.

Since (B.6) can be rewritten as:

$$\left(\frac{R_S^1 - R_S^0}{R_S^0}\right)(1 + i_F) = i_H - i_F, \qquad \text{(B.7)}$$

or,

$$\left(\frac{R_S^1 - R_S^0}{R_S^0}\right) + \left(\frac{R_S^1 - R_S^0}{R_S^0}\right)(i_F) = i_H - i_F, \qquad \text{(B.8)}$$

and since the second term in the lefthand side of (B.7) is negligible (due to the notion of the 'second order of small'), (B.6) [or (B.7)] can be re-expressed in approximate form as follows[2]:

$$\frac{R_S^1 - R_S^0}{R_S^0} = i_H - i_F \qquad \text{(B.7*)}$$

That is, the country with the higher (lower) inflation rate will experience its currency depreciation (appreciation) and the rate of depreciation (appreciation) approximately equals its inflation rate differential from that of the other country. Take a look at Figure 6.3 at this point. Here we measure currency appreciation (or depreciation) along the vertical axis, and the horizontal axis measures the inflation differential between two counytries. Figure 6.3(a) shows exact picture with non-negligible order of small case, and Figure 6.3(b) exhibits the situation with negligible order of small. The international commodity arbitrage that brings about purchasing power parity signifies that a country's percentage change in exchange rate and its inflation rate in relation to another country should be so aligned that these should stay on the 45° line.

Figure 6.3

---

[2] More simply, if you take the logarithmic transformation on (B.1), ignoring time subscript such as 0, and then differentiate, you get: $dR_s/R_s = dP_H/P_H - dP_F/P_F$. dR_s/R_s is the measure of currency appreciation (or depreciation), dP_H/P_H the measure of home country's rate of inflation (i_H), and dP_F/P_F the measure of foreign country's rate of inflation (i_F). Thus we establish once again that currency appreciation (or depreciation) equals the inflation rate differential.

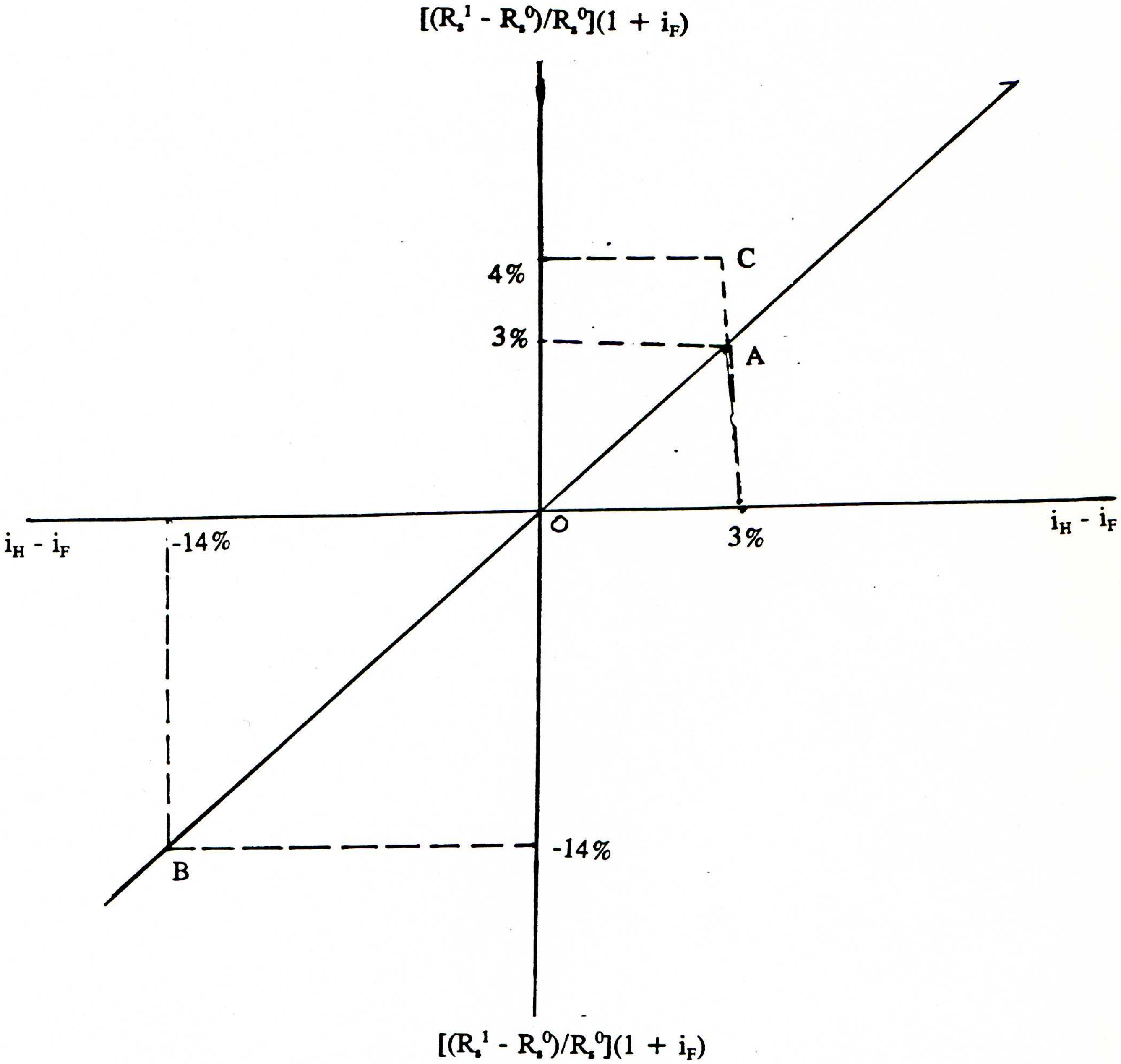


Figure 6.3(a)

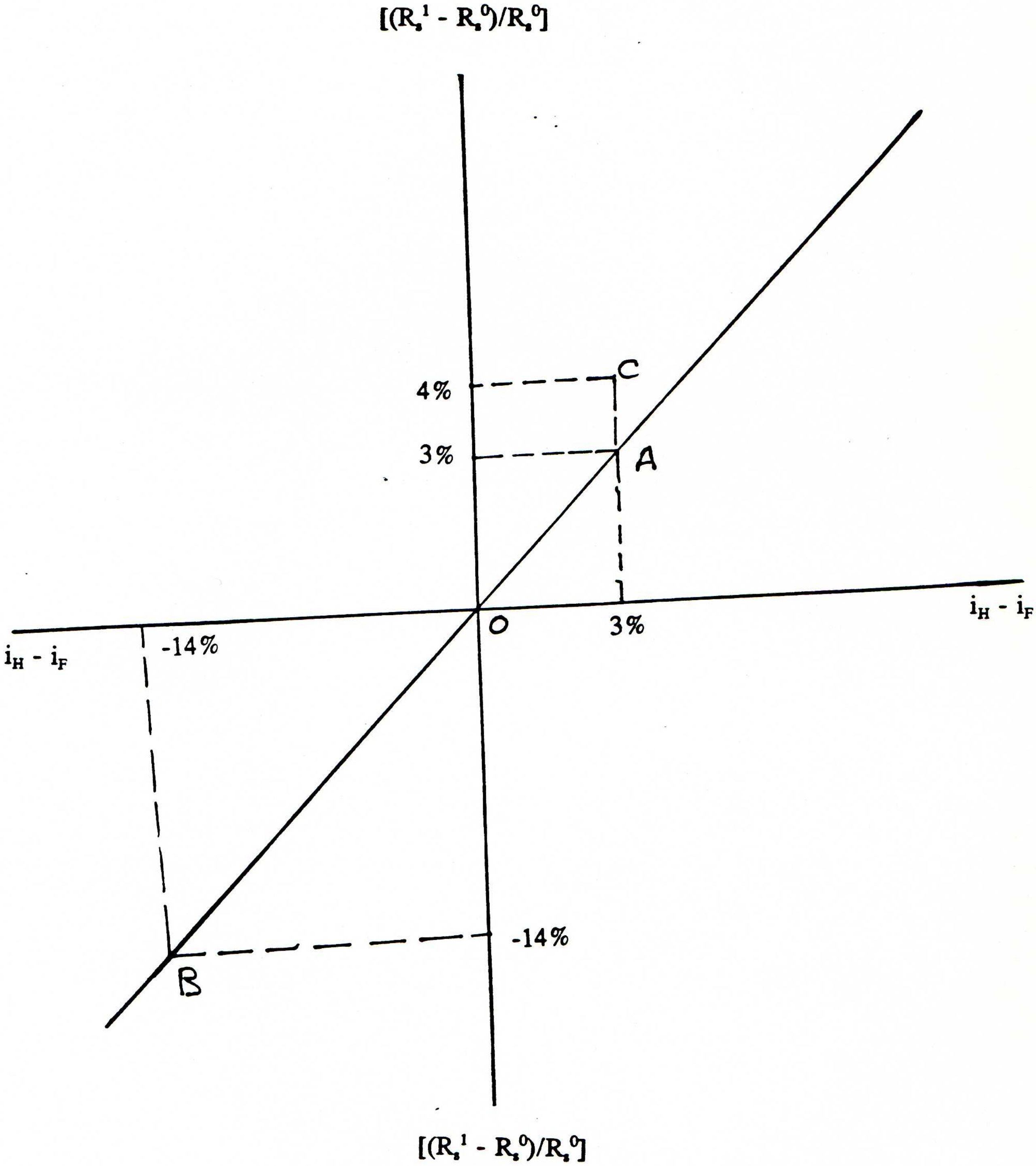


Figure 6.3(b)

Suppose British inflation rate is 3 percent below U.S. inflation rate (say, for example, the U.S inflation = 6% and the British inflation rate = 3%). In this situation, as coorrdinates of A shows in condition of purchasing power parity in its relative version, British currency, that is pound sterling will have 2.83 percent appreciation against U.S. dollars (in non-negligible small order case, Figure 6.3(a)) or 3 percent appreciation against U.S. dollars (in negligible small order case, Figure 6.3(b)). Point B on this diagram shows that when Mexican inflation rate is 20 percent while the American inflation rate is 6 percent, Mexican peso depreciates by 11.67% (in non-negligible small order case, Figure 6.3(a) or by 14% (in negligible small order case, Figure 6.3(b))[3].

In reality, that is not always the picture. If, for instance, you note a point, say, C in Figure 6.3, what would that mean to you? A moments's reflection should reveal that international commodity arbitrage has not established the law of one price across the countries at this point of observation. But since forces are on, the law of one price will tend to reduce home country inflation rate, raise foreign country inflation rate, and thus trigger a drop in the foreign country's currency appreciation. Other possible scenarios are left for interested readers.

---

[3]. In the non-negligible case of small order, $\{(R_s^1 - R_s^0)/R_s^0\}(1 + i_F) = 3\%$, and hence $\{(R_s^1 - R_s^0)/R_s^0\} = 0.03/1.06 = 2.83$ percent. In the Mexican case, it is $0.14/1.2 = 11.67$ percent.

How Valid is the Purchasing Power Parity?

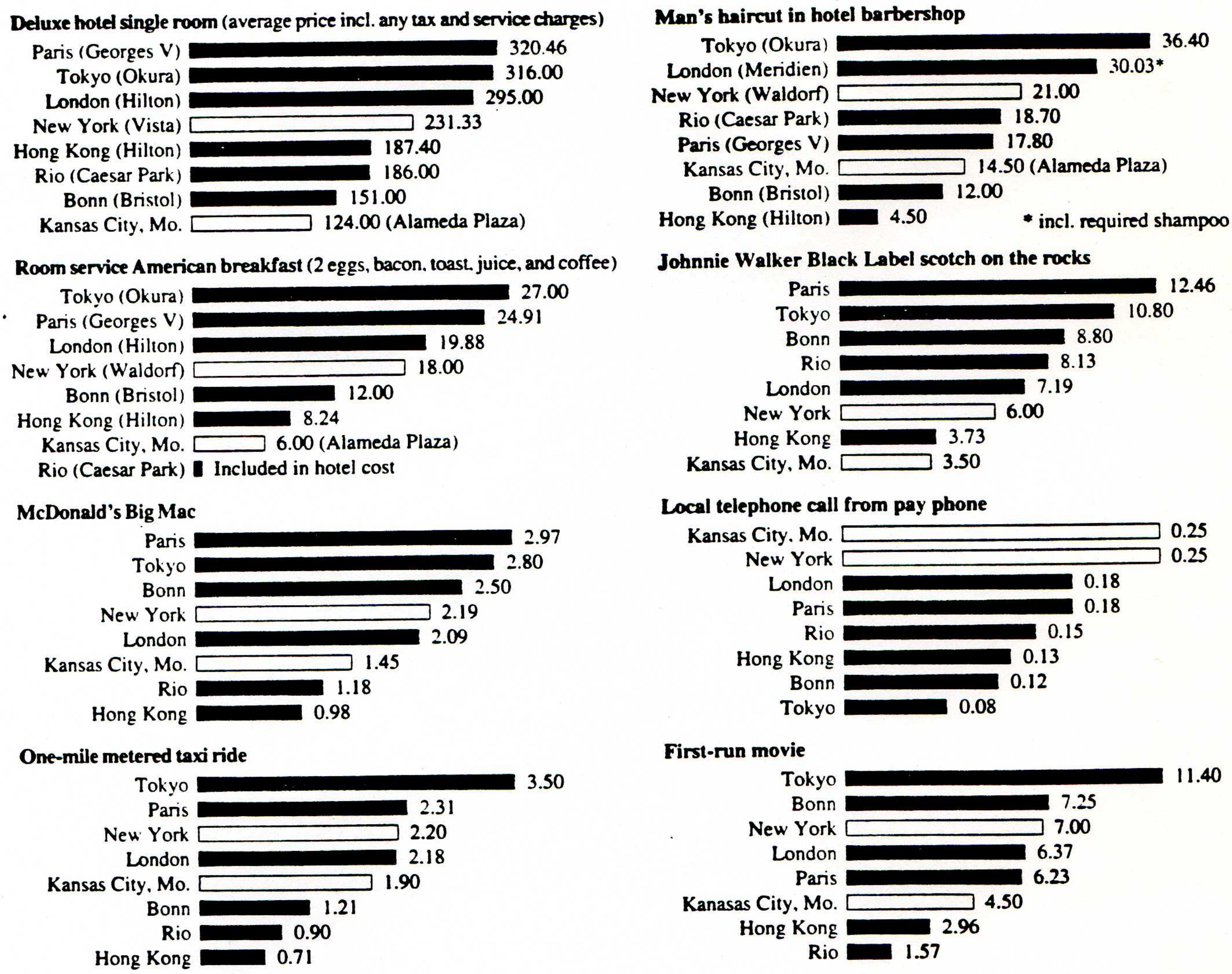
At the outset it has been noted that if perfect competition **prevails**, and **no** market distortions, trade barriers such as tariffs and taxes, transport costs, and the like exist, then the dynamics of arbitrage, - that is, buying cheap and selling dear determine the eventual (equilibrium) equality of each country's purchasing strength; exchange rate then becomes the ratio of home price and foreign price of the same commodity. So, it appears clear that if two markets are imperfectly competitive, then purchasing power parity fails to exist. It is possible then that the same automobile may sell for £20,000 in London and for $41,280 in New York at the same point in time even though at that point exchange rate is 2. This price differential of $1,280 ($\equiv$ 41,280 - 20,000 x 2) may be due to the existence of transport costs or may be owing to the lack of (or recognition of) this price differential by American customers.

Let us look at the price structure of some of the commodities across nations. Here is a table of data on such price structure in different countries:

Table B.1

Why do you note so much of deviation? Is it simply spurious or more fundamental? Several analyses have been conducted to examine the validity of puchasing power parity, and many results are already in to support or oppose the validity of this theory. It has been stated that purchasing power parity holds in the long run, but it does hardly work in the short run, and

Comparison of Prices of Nontraded Goods and Services



"WHat the Dollar Won't Buy", The World Street Journal, December 4, 1987

Table B.1

empirically the finding seems to be in order. However, beyond empirical evidence, when you examine purchasing power parity you note immediately that P_H or P_F is hardly defined correctly. If these country-wide prices were for one (internationally) tradable good such as Jaguar and commodity arbitrage across nation were set in motion, purchasing power parity would be true. But, in reality, P_H is the price index of the home country with large number of goods and services a set of which consists of internationally non-traded commodities, and so is the price index of the foreign country P_F. Under this condition, purchasing power parity will not hold. The Table B.1 makes this point loud and clear.

Real and Effective Rates of Foreign Exchange:

Two other concepts or measures of foreign exchange are often used in the literature, and they are (i) ***real rate of foreign exchange***, and (ii) ***effective rate of foreign exchange***. Real (spot) rate of foreign exchange at time t ($R_s^{t(R)}$) is nothing but the inflation-adjusted nominal (spot) rate of exchange. It is defined as follows:

$$R_s^{t(R)} = R_s^t.\{(1 + i_F)/(1 + i_H)$$

where i_F and i_H denote foreign and domestic (home) rates of inflation at time t, and R_s^t is the nominal rate of foreign exchange at time t. Earlier we have just called it spot rate of exchange without characterizing it as *nominal*. It is obvious then that if purchasing power parity in its relative version holds, that is, if:

$$R_s^t/R_s^o = (1 + i_H)/(1 + i_F), \text{ then } R_s^{t(R)} = R_s^0.$$

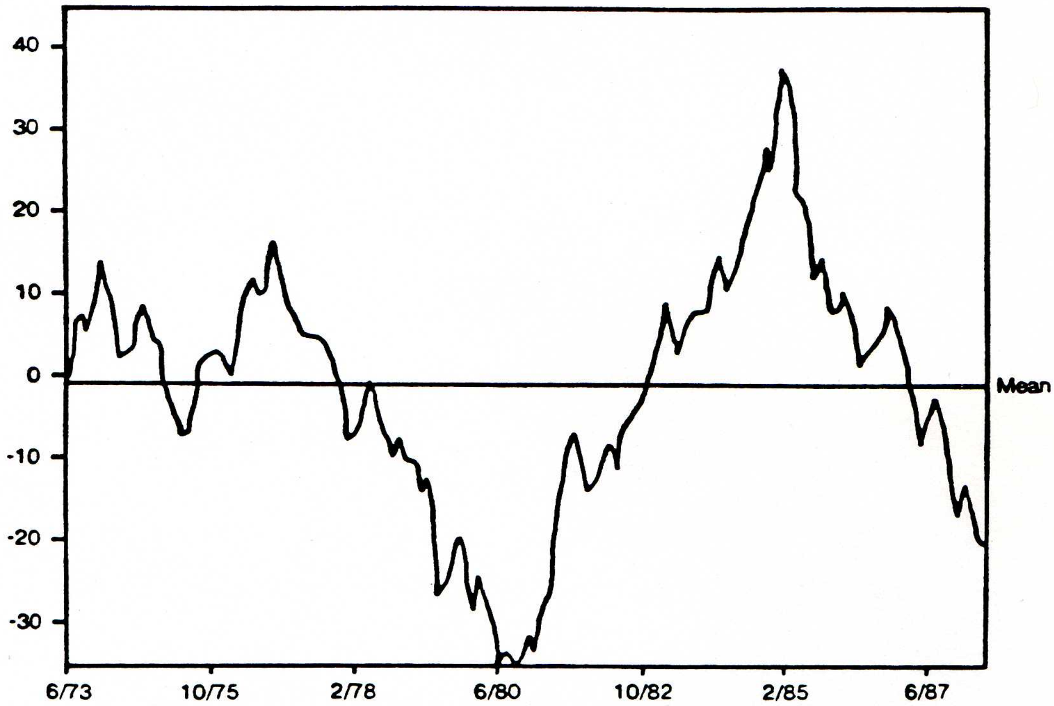
That means, real exchange rate at time t is exactly equal to current nominal exchange rate. At this point, it is instructive that we take a look at two pictures of real exchange rates. Figures 6.4(a) and 6.4(b) exhibit real exchange rates of U.K and Japan.

Figure 6.4

Effective rate of foreign exchange for a currency is a measure of that currency's trade-weighted average appreciation or depreciation *vis-a-vis* the currencies of other major countries. The weights reflect the relative significance of of the major trading partners in a country's trade. The weights are based on country's bilateral and multilateral shares of trade in manufactures and they also take into account the relative size of the countries' economies. A more appropriate measure of exchange rate is ***real effective rate of foreign exchange***. It is the index of effective exchange rate adjusted for inflation differential. Table B.2 presents effective and real effective exchange rates for some of the countries of the world over a period of time.

Table B.2

Real Exchange Rate: United Kingdom
(June 1973 to May 1988)

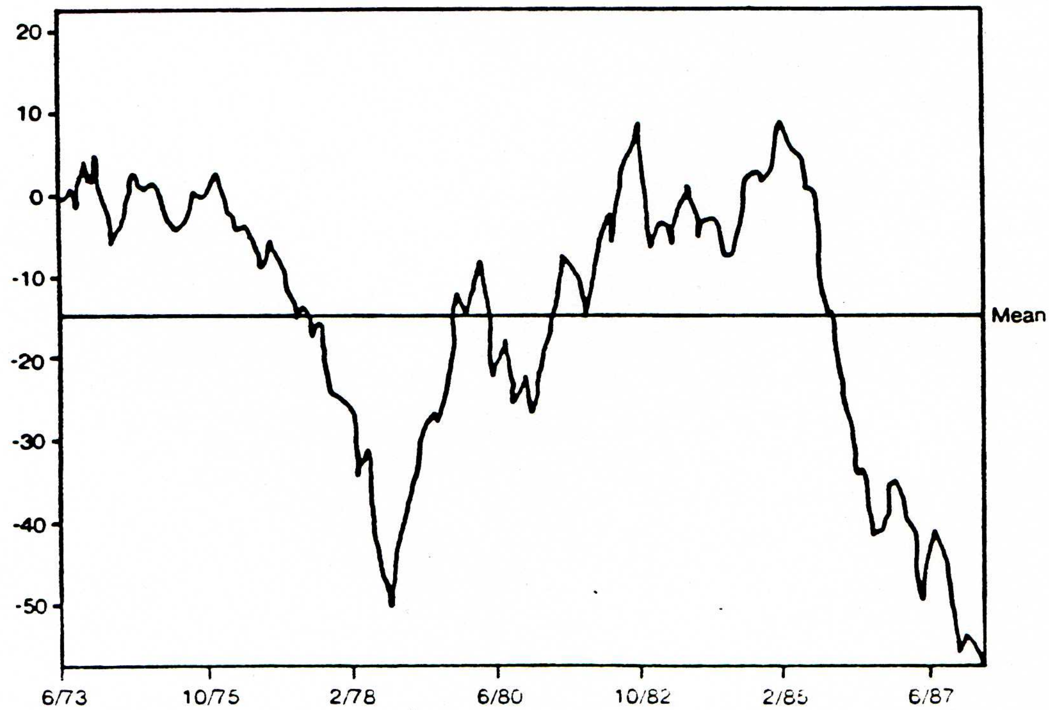


The levels of the real pound/dollar exchange rate reflect the sharp dollar depreciation of the late 1970s, the even greater appreciation of the early 1980s, and the decline since February 1985.

Source for all charts: Computed at the Federal Reserve Bank of Atlanta from data published in *International Financial Statistics*.

Figure 6.4(a)

Real Exchange Rate: Japan
(June 1973 to May 1988)



Over the entire period, the dollar declined 57 percent against the yen—more than against any of the other currencies.

Figure 6.4(b)

Table B.2

EFFECTIVE AND REAL EFFECTIVE EXCHANGE RATES

	United States	Canada	Japan	United Kingdom	West Germany	France	Italy	Belgium	The Netherlands	Switzerland	Austria	Denmark	Norway	Sweden	Australia	Spain
EFFECTIVE EXCHANGE RATES																
Pre-June 1970 parities	118.5	94.9	79.3	116.2	91.6	97.5	110.9	95.9	98.2	88.3	97.8	97.9	98.1	102.2	92.2	98.4
Smithsonian central rates	107.0	101.3	88.7	116.0	95.4	96.1	108.3	97.0	99.1	91.6	98.0	96.0	96.3	100.5	91.7	98.1
1981																
May	107.3	83.4	122.2	84.8	132.6	84.8	48.3	106.3	113.3	160.8	119.5	84.7	105.7	90.9	80.3	66.2
June	109.3	83.6	121.9	82.1	131.9	85.0	47.9	105.8	113.1	165.6	119.0	84.6	104.6	91.1	82.1	65.6
July	111.6	83.6	118.8	79.5	131.6	85.0	47.9	105.6	113.1	167.9	119.4	85.0	104.3	91.2	84.4	65.1
August	113.3	83.0	119.4	78.7	131.3	84.2	47.9	105.8	113.2	165.9	119.7	84.6	105.2	91.1	85.2	64.7
September	110.3	84.2	119.4	75.4	133.8	85.6	47.9	106.9	115.0	170.9	121.1	87.6	106.5	85.4	84.5	64.9
October	109.7	83.9	117.4	75.3	137.4	83.7	46.5	106.8	117.9	179.8	123.0	88.1	106.0	81.8	83.7	64.0
November	107.6	84.7	121.1	76.9	137.2	82.8	46.1	106.2	118.8	187.7	122.4	87.5	106.2	81.5	82.2	63.7
December	107.7	84.9	123.9	77.6	137.0	82.4	46.0	104.9	118.8	186.4	122.0	86.4	107.7	81.3	81.2	63.2
1982																
January	109.2	84.6	121.5	78.0	137.0	81.9	45.9	104.6	118.7	186.2	122.2	85.6	107.8	81.4	80.8	63.2
February	112.3	83.5	117.6	78.5	136.8	81.8	45.9	102.0	118.8	186.8	121.9	84.5	108.8	81.6	80.7	63.3
March	114.1	83.3	115.6	78.0	139.1	81.9	45.6	95.8	120.6	190.0	122.4	83.1	109.6	81.9	80.1	61.9
April	115.2	83.2	115.0	77.5	140.4	81.6	45.2	95.1	120.3	185.0	123.1	82.9	110.0	81.8	80.3	61.8
2	115.6	83.0	113.9	78.1	139.9	81.7	45.2	94.9	120.2	187.9	122.8	82.4	109.8	81.7	80.4	61.5
9	116.0	83.0	113.6	77.3	140.0	81.6	45.4	95.0	120.5	185.3	123.4	82.8	110.1	81.9	80.6	61.7
16	115.9	83.3	113.7	77.5	140.0	81.6	45.2	95.0	120.3	185.4	123.2	82.9	110.2	81.7	80.5	61.8
23	115.0	83.4	115.5	77.5	140.4	81.5	45.1	95.1	120.2	185.2	123.0	83.0	110.0	81.8	80.2	62.0
30	113.8	83.1	117.8	77.1	141.0	81.8	45.1	95.6	120.7	182.8	123.3	83.4	109.6	81.8	79.7	62.0
May 7	112.9	83.0	118.3	77.4	141.2	82.2	45.1	95.7	120.7	183.2	123.5	83.9	109.0	81.6	79.4	62.0
14	112.9	82.0	118.3	77.7	141.8	82.3	45.2	95.7	120.8	181.9	123.8	84.2	109.1	81.4	79.1	62.1
21	113.7	82.1	117.2	77.0	141.7	82.4	45.3	95.8	120.9	179.6	123.9	84.1	109.3	81.4	79.2	62.2
REAL EFFECTIVE EXCHANGE RATES																
1978	96.3	92.1	106.9	106.2	103.4	97.3	90.8	98.5	104.0	123.9	109.0	102.7	98.0	96.4	89.4	97.1
1979	96.4	92.2	95.7	118.6	104.1	99.3	90.6	96.0	100.6	117.1	107.8	100.8	92.7	99.1	88.3	109.2
1980	98.1	91.3	93.5	138.1	101.1	101.1	93.0	90.8	97.3	106.8	108.5	93.4	94.9	100.6	89.7	103.4
1981	108.7	91.4	94.3	140.9	96.6	98.9	91.5	85.0	93.0	105.5	106.0	89.1	99.1	100.3	98.8	97.6
May	109.9	90.8	94.6	146.3	95.7	97.1	91.7	85.3	91.7	101.1	105.9	88.1	101.1	104.1	97.4	99.7
June	111.7	91.4	94.0	142.1	95.2	97.8	91.1	84.8	91.2	104.0	105.0	88.8	99.9	103.6	100.1	98.4
July	113.5	91.7	90.9	137.1	94.9	99.4	90.9	84.2	91.3	105.0	105.0	88.5	98.7	104.7	103.4	98.6
August	114.4	91.6	91.6	135.9	95.0	98.6	91.1	84.1	91.4	103.9	105.0	87.9	98.8	104.5	104.4	98.5
September	110.7	93.1	91.5	130.4	96.4	101.3	91.8	85.0	92.5	106.6	105.6	90.7	100.6	98.9	103.7	98.8
October	111.4	92.2	89.0	130.8	99.3	97.3	90.2	84.9	94.5	111.0	107.3	91.3	99.3	95.8	102.5	98.0
November	109.5	92.5	91.4	133.7	98.7	97.8	89.8	84.3	94.6	114.9	106.3	90.4	99.5	95.3	100.8	97.4
December	109.2	93.0	93.4	135.1	98.5	97.5	90.0	83.8	94.0	113.8	105.9	89.6	100.2	95.3	99.5	97.8
1982																
January	110.7	92.7	90.7	136.0	98.3	97.6	90.2	81.2	95.9	112.5	106.4	87.7	102.3	96.9	99.7	97.6
February	113.1	92.2	87.7	137.1	97.5	97.9	90.7	80.6	94.5	112.4	106.6	86.7	103.7	97.6	99.7	98.5
March	114.4	92.3	85.9	136.2	98.5	98.8	90.2	76.2	95.9	114.0	107.6	85.6	105.6	98.2	99.4	97.0
April	115.7	92.1	84.9	135.4	99.5	99.2	90.0	74.5	95.5	110.7	108.1	84.8	105.6	99.3	99.6	96.9

Source. Morgan Guaranty Trust Company, *World Financial Markets*, May (1982).

Note. Index numbers, March 1973 = 100. The index of the effective exchange rate for a currency is a measure of that currency's trade-weightec average appreciation or depreciation vis-à-vis the currencies of 15 other major countries. The index of the real effective exchange rate is the index o the effective exchange rate adjusted for inflation differentials, which are measured by wholesale prices of nonfood manufactures. The exchange rate: used in the construction of this index are the averages of daily noon spot exchange rates in New York for months and for weeks ending on date: shown. The trade weights used are based on 1976 bilateral trade in manufactures. Annual figures are averages of months.

C. THE FISHER OPEN PARITY

Irving Fisher established the following relation between nominal and real interest rate *via* inflation rate:

$$1 + r = (1 + \rho)(1 + i) \qquad \text{(C.1)}$$

where r ≡ nominal (that is, money) rate of interest, ρ ≡ real rate of interest, and i ≡ inflation rate. Relation (C.1) equals:

$$r = \rho + i \qquad \text{(C.2)}$$

as $\rho.i \approx 0$ (second order of small). Now, (C.2) can be be written for the home and the foreign country as follows:

$$r_H = \rho_H + i_H \qquad \text{(C.3.1)}$$

$$r_F = \rho_F + i_F \qquad \text{(C.3.2)}$$

It has been established in pure theory of international trade that real rate of interest across countries are equal, and that means $\rho_H = \rho_F$. Under this equality of real rates of interest across nations then one can get from (C.3.1) and (C.3.2) (by subtracting (C.3.2) from (C.3.2)):

$$r_H - r_F = i_H - i_F \tag{C.4}$$

That is, **nominal interest rate differential equals the inflation rate differential between two countries**. This is the Fisher Relation across nations (call it **Fisher Parity**). Figure 6.4 exhibits this Fisher Parity. In the case where second order small is considered as negligible (Figure 6.4(a)), interest rate differential matches exactly inflation rate differential between two countries exactly. 45° line (passing through the origin) traces this exact correspondence. If $\rho.i_H$ and $\rho.i_H$ (where ρ is the real interest rate in both countries) are not negligible, Fisher Relation should read as follows:

$$r_H - r_F = i_H - i_F + \rho(i_H - i_F) \tag{C.5}$$

and its diagrammatic depiction is given by Figure 6.4(b). Here also the slope is 45°, but the intercept is $-\rho(i_H - i_F)$. The 45^0 line AB represents (C.5) if $-\rho(i_H - i_F) > 0$; CD is the graph of (C.5) if $-\rho(i_H - i_F) < 0$.

In real life what do all these convey? (In the case of where second order of small means zero), if home country's inflation rate exceeds foreign country's inflation rate by, say, 5 percent, then home country's nominal interest rate is exactly higher by that 5 percent; if second order small is not ignored, then home excess inflation rate over foreign inflation rate varies by the same magnitude of home country's excess interest rate over foreign country's interest rate *less* inflation rate differential *times* real rate of interest $\rho(r_H - r_F)$.

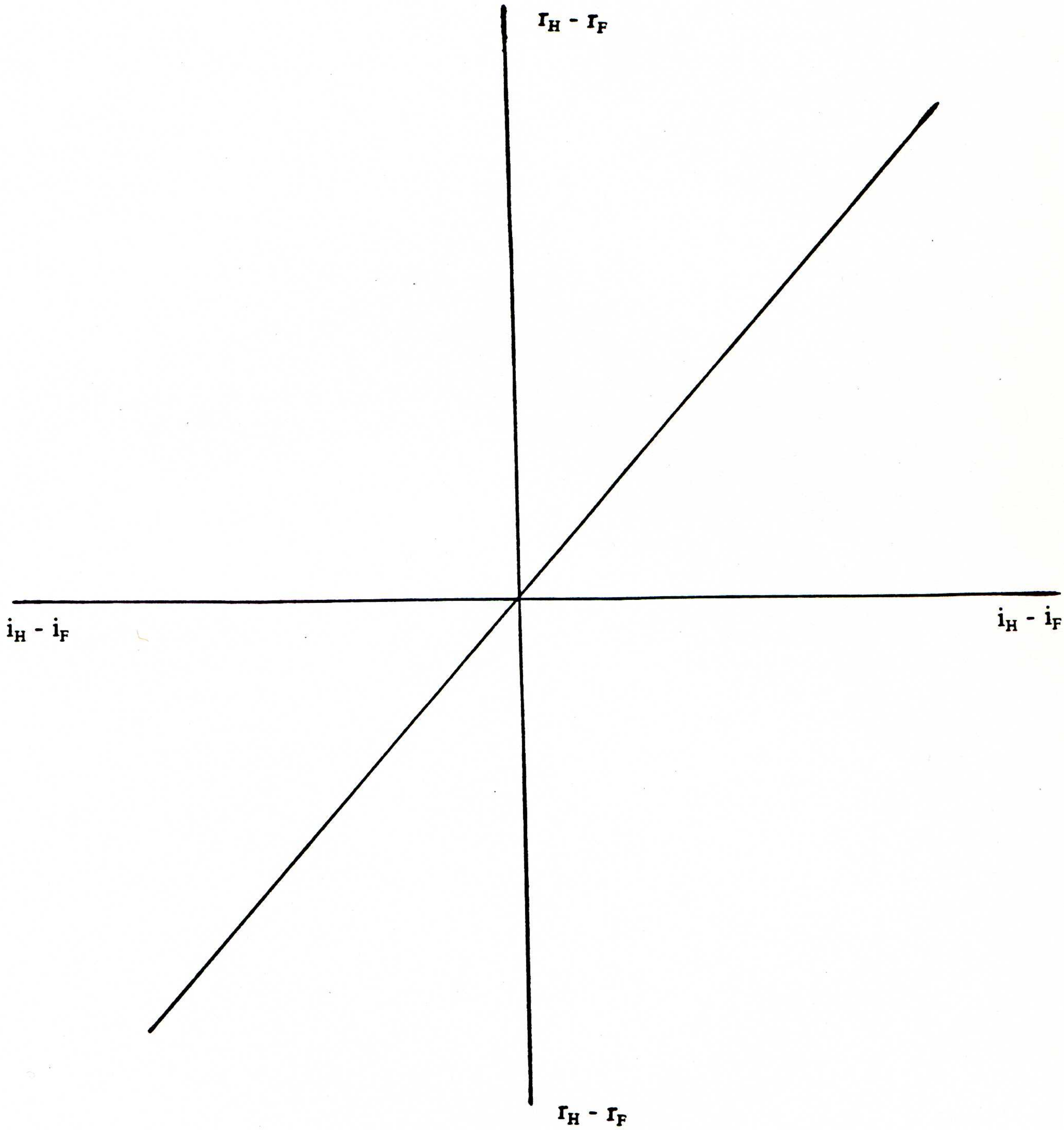


Figure 6.4(a)

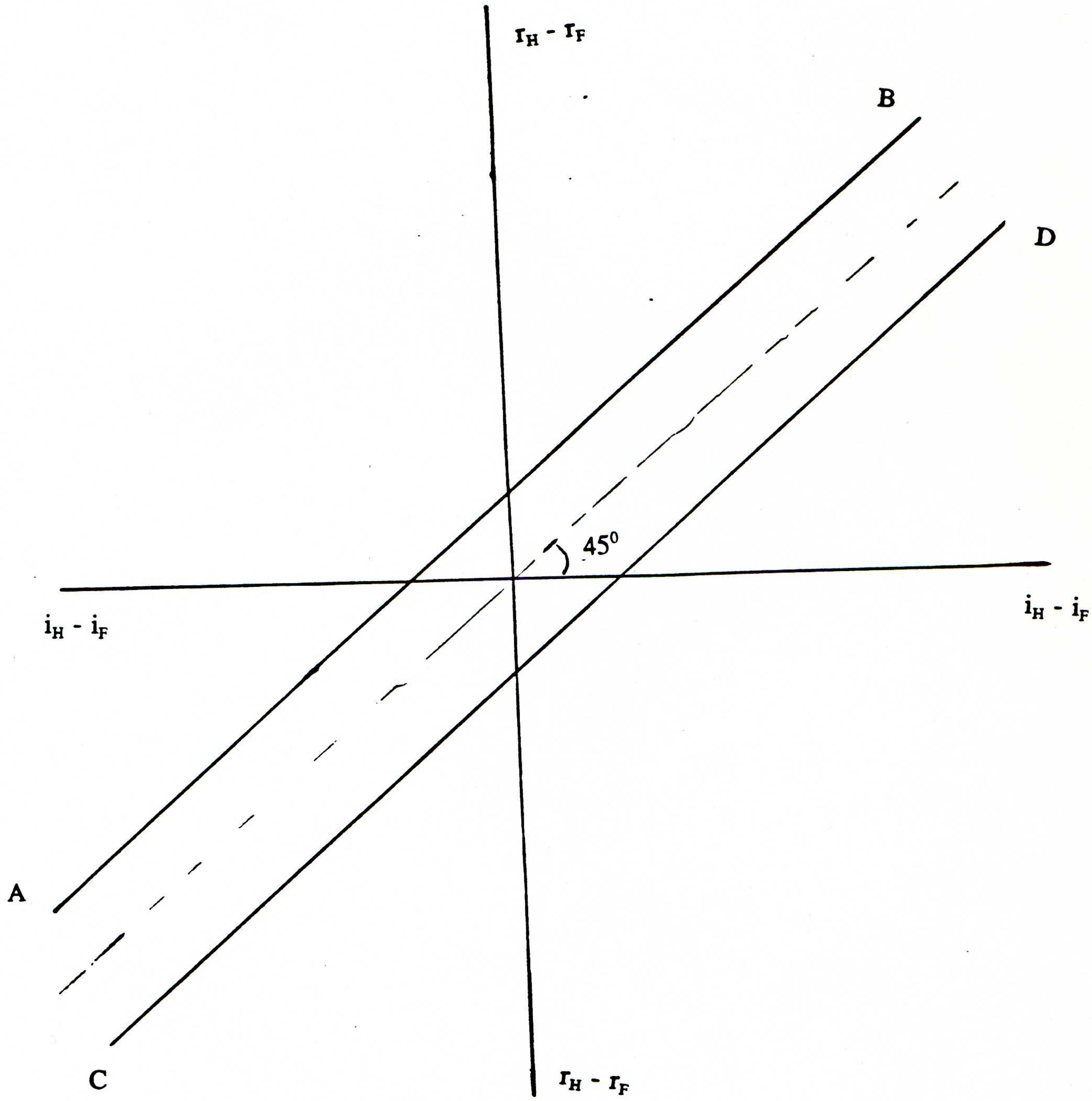


Figure 6.4(b)

Now try to integrate purchasing power parity into the Fisher relation developed thus far. One may note that (B.7) or (B.8) being inserted into (C.4) gives the following expressions:

$$\left(\frac{R_S^1 - R_S^0}{R_S^0}\right)(1 + i_F) = \frac{i_H - i_F}{1 + i_F^1} = r_H - r_F \tag{C.6}$$

and

$$(R_s^1 - R_s^0)/R_s^0 = r_H - r_F \tag{C.7}$$

These are the expressions of ***Fisher Open Principle*** (or ***Fisher Open Parity***). These expressions state that interest rate differential between two countries equals rate of change in foreign exchanhge rate (currency appreciation or depreciation). If Mexico's interest rate exceeds U.S. interest rate by 14 percent, then Mexican Peso is expected to depreciate by 14 percent (if (C.6) holds; 11.67 percent if (C.7) holds). If you replace forward rate (R_f^0) by future spot rate (expected one year from now) (R_s^1) in Figures 6.3(a) and 6.3(b), you find the pictures of Fisher Open Parity or the deviation thereof. 45^0 line portrays Fisher Open Parity, and any co-ordinates above and below this 45^0 line define deviations from the generalized Fisherian equilibrium.

D. THE FORWARD FUTURE-SPOT PARITY

It is contended that forward rate is the unbiased predictor of future spot rate of exchange. That means if one-year forward rate is 2.15 today, expected spot rate of exchange one year from today is 2.15. Note - it is the ***expected*** - not necessarily the actual spot rate a year from now is 2.15. In mathematical expression, it is as follows:

$$R_f^0 = R_s^1 + \epsilon \qquad (D.1)$$

where R_s^1 denotes spot rate one year from today (unknown), and ϵ is a nuisance term (white noise) whose expected value is zero. That means:

$$R_s^0 = E(R_s^1) \qquad (D.2)$$

If market is truly *efficient*, the principle of arbitrage should yield this result. Many empirical studies show the validity of this result, and yet many other researchers find no truth in this parity at all. We will leave it in the hands of interested students to reexamine this parity in light of available evidence.

ARE THESE PARITIES INTERRELATED?

It is instructive to see if these parities are related to each other. From the **Interest Rate Parity**, you have:

$$(R_f^0 - R_s^0)/R_s^0 = r_H - r_F \qquad (1)$$

From the **Purchasing Power Parity** one gets:

$$(R_s^1 - R_s^0)/R_s^0 = i_H - i_F \tag{2}$$

Next, **Fisher Relation** and **Fisher Open Parity** yield for us:

$$r_H - r_F = i_H - i_F \tag{3}$$

and

$$(R_s^1 - R_s^0)/R_s^0 = r_H - r_F \tag{4}$$

and finally from the **Forward Future-Spot Parity** has come out the following:

$$R_f^0 = R_s^1 \tag{5}$$

At this point, one finds it easily that if any three of these parities hold, then the remaining ones must hold. Note, for instance, that when (1), (2), and (3) hold, (5) emerges automatically; if (5), (3), and (2) hold, (4) must be true, and so on and so forth. In these exercises, approximate versions have been considered, but if exact expressions of these parities are taken into account, the result remains unscathed.